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STATISTICAL DESCRIPTION OF WAVE INDUCED VIBRATORY STRESSES IN S--ETC(U)

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STATISTICAL DESCRIPTION OF WAVE INDUCED VIBRATORY STRESSES IN SHIPS

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Sverre Gran

Det norske Veritas

DECEMBER 1980
FINAL REPORT

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PREPARED FOR

U.S. DEPARTMENT OF TRANSPORTATION
UNITED STATES COAST GUARD
OFFICE OF MERCHANT MARINE SAFETY
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NOTICE

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16. Abstract

This report contains general, theoretical considerations of two peak response spectra with particular attention to wave induced bending and springing stresses in ships.

Relationships between periods, RMS values and spectral width are discussed.

Different representations of the resulting extreme value under stationary conditions are considered.

An approach to the long term description and prediction of extreme stresses has been developed, and has in part been compared with alternative methods. Empirical data related to the theoretical deductions have been attached in an appendix.

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Summary

This report contains general, theoretical considerations of two-peak response spectra with particular attention to wave induced bending and springing stresses in ships.

Relationships between periods, RMS-values and spectral width are discussed.

Different representations of the resulting extreme value under stationary conditions are considered.

Fatigue contributions from the respective spectral components are evaluated and discussed.

An approach to the long term description and prediction of extreme stresses has been developed, and has in parts been compared with alternative methods. Empirical data related to the theoretical deductions have been attached in an appendix.

4 Indexing terms

- SPRINGING
- SPECTRAL ANALYSIS
- STATISTICAL ANALYSIS
- EXTREME CONDITIONS

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1. SUMMARY, ASSUMPTIONS AND CONCLUSIONS.

1.1 A preliminary survey

The present report is concerned with random processes, the power spectra of which consist of two distinct peaks. In marine engineering such processes are found in ships and slender off-shore structures who exhibit a resonant random vibration in addition to the semi-static forces excerted by the passing waves.

With reference to ships the resonant stress is termed springing and corresponds to the two node mode of vibration. The semi-static response is termed bending and corresponds to the more familiar hogging and sagging stresses.

The main objective of the work is to investigate how the extreme stress in both a short and a long time interval is influenced by the mixing ratio of bending and springing.

In the short term case, i.e. under stationary conditions, the stress peaks are supposed to follow a Rice Probability distribution, which is common for signals of arbitrary spectral shape. The distribution parameters, i.e. the RMS and the spectral width in this case are simple, algebraic functions of the individual RMS and periods for springing and bending.
Once the Rice distribution of peaks is adopted, the short term extreme stress is also known, both in terms of the characteristic value and the probability distribution. As the exact extreme value expressions are fairly unsurveyable there are a number of approximate formulae which give more direct insight into uncertainty, effect of period estimate etc.
The long term case is heavily based on certain properties of the generalized gamma functions.

It has been pointed out that there is a logical relationship between the Rice and the general gamma distributions.(Section 3.3) By short term gamma distributed peaks and long term gamma distributed RMS, a method has been pointed out to establish a long term gamma distribution of stress peaks. Once this is established, the long term extreme may be evaluated.

At the present stage, the short term stationary condition case is fairly complete. The long term case is less complete from a theoretical point of view, but the evaluation methods pointed out should give reasonably correct results when applied to practical problems. For this purpose a table of the functions required are included in Appendix A.
Although not a part of the original project, some evaluations of fatigue life has been included, partly for the sake of completeness, and partly because it is felt that additional vibration components may be more important for deterioration processes than for direct overloading.

Some empirical material from a measuring project on a tanker has been included in Appendix B. This material has been presented in a fashion which conforms with the theoretical work. An evaluation of the measured data against the theoretical results has, however, not been undertaken. Previous documentation of the measurements are found in /14/.

1.2 Basic assumptions.

The work and conclusions in this report are based on the following assumptions:

A When considered separately the springing and bending stress components are gaussian, narrow-banded random processes. This implies that the average zero-up-crossing period is equal to the average peak period, and that the amplitudes are Rayleigh distributed.

B The bending and springing stress components are statistically independent under stationary conditions. This implies that the resulting stress process is a gaussian broad-banded process with Rice distributed maxima.

C The RMS-values of the springing and bending stress components are statistically independent in the long run.

D There is a high degree of independence between the total RMS-value and the variables connected with the average periods, such as the spectral width parameter.

E The contributions to crack propagation velocity or fatigue rate from bending and springing are to be added together linearly.

Assumption A is supported by Fig.1,2.1.

Assumption B has not been extensively testes within the present project or in previous related projects. In case there is a positive correlation between the instantaneouse bending and springing stresses, the present theories will underestimate the total stress extremes.

Assumption C is supported by Table 1.2.1.

Assumption D is supported by Table 1.2.2.

Assumption E is to some degree discussed in the text in Chapter 8.

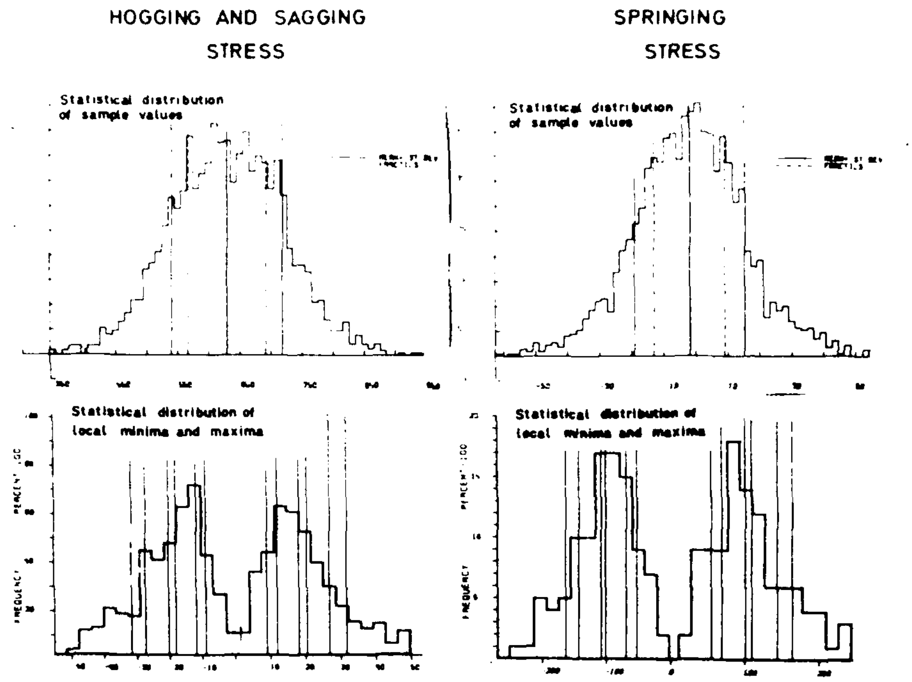

Fig.1.2.1 Statistical properties of bending and springing stress components within a short term record. The sample values are gaussian and the amplitudes are Rayleigh-like. From /14/.

(ALL CONDITIONS)

A2.4 CORRELATION BETWEEN BENDING AND SPRINGING STRESS LEVEL

IN THE PRESENT SECTION THE CORRELATION BETWEEN THE SPRINGING STRESS AND THE BENDING STRESS LEVEL WILL BE MORE CLOSELY INVESTIGATED.

CONCERN STEMS FROM THE FACT THAT SPRINGING STRESS LEVEL UNDER GIVEN ENVIRONMENTAL CONDITIONS IS DIFFICULT TO PREDICT, WHILE PREDICTION OF BENDING STRESS CAN BE PERFORMED WITH REASONABLE ACCURACY BY A NUMBER OF METHODS.
THEREFORE, IF THERE IS A CERTAIN CORRELATION BETWEEN THESE VARIABLES, THE SPRINGING STRESS IN A GIVEN CONDITION CAN BE PREDICTED ON THE BASIS OF THE BENDING STRESS PREDICTION.

THE INVESTIGATION IS BASED ON 1119 OBSERVATIONS.

SOME MAIN DATA OF THE OBSERVATIONS ARE LISTED IN THE FOLLOWING TABLE :

AVERAGE BENDING ROOT-E VALUE	38.60 KP/CM**2
STANDARD DEVIATION OF BENDING ROOT-E VALUE	33.41 KP/CM**2
LARGEST VALUE OF THE BENDING ROOT-E VALUE	217.00 KP/CM**2
CORRESPONDING VALUE OF SPRINGING ROOT-E VALUE	44.00 KP/CM**2
SMALLEST VALUE OF THE BENDING ROOT-E VALUE	2.00 KP/CM**2
CORRESPONDING VALUE OF SPRINGING ROOT-E VALUE	1.00 KP/CM**2
AVERAGE SPRINGING ROOT-E VALUE	13.49 KP/CM**2
STANDARD DEVIATION OF SPRINGING ROOT-E VALUE	13.22 KP/CM**2
LARGEST VALUE OF THE SPRINGING ROOT-E VALUE	97.00 KP/CM**2
CORRESPONDING VALUE OF BENDING ROOT-E VALUE	46.00 KP/CM**2
SMALLEST VALUE OF THE SPRINGING ROOT-E VALUE	1.00 KP/CM**2
CORRESPONDING VALUE OF BENDING ROOT-E VALUE	7.00 KP/CM**2

POSSIBLE LINEAR REGRESSION CURVES ARE :

$$X = 22.944 + 1.161 * Y$$

$$Y = 6.465 + .182 * X$$

WHERE X IS THE BENDING ROOT-E VALUE IN KP/CM**2
AND Y IS THE SPRINGING ROOT-E VALUE IN KP/CM**2

THE COEFFICIENT OF CORRELATION IS .459

IF Y IS KNOWN, THE UNCERTAINTY IN X IS REDUCED FROM 33.41 (THE STANDARD DEVIATION IN THE TABLE ABOVE) TO 29.67 BY USING THE FIRST REGRESSION RELATION.
SIMILARLY, IF X IS KNOWN, THE UNCERTAINTY IN Y IS REDUCED FROM 13.22 TO 11.75 BY USING THE SECOND RELATION.
AN INTUITIVE IMPRESSION OF THE CORRELATION IS ALSO OBTAINED BY INSPECTION OF THE SCATTER DIAGRAM ON THE NEXT PAGE.

SCATTER DIAGRAM (ALL CONDITIONS)

THIS DIAGRAM SHOWS THE SIMULTANEOUS DISTRIBUTION OF OBSERVATIONS AND THE PARAMETERS OF THE CLASS-WISE AND MARGINAL DISTRIBUTIONS :

X: BENDING ROOT-E VALUE IN KP/CM**2 Y: SPRINGING ROOT-E VALUE IN KP/CM**2

CLASS MIDPOINT			X:	10.00	30.00	50.00	70.00	90.00	110.00	130.00	150.00	170.00	190.00	210.00	TOTAL
	AVERAGE VALUE		Y:	8.20	12.27	17.52	22.16	25.88	26.25	27.73	28.08	29.17	33.00	55.00	14.28
		STANDARD DEVIATION	Y:	7.03	10.05	14.68	16.73	18.37	13.64	18.14	15.88	13.82	23.15	10.00	13.23
Y:	X:	X:													
5.00	28.39	22.39		250	203	95	20	10	3	1	1	1	1	0	585
15.00	41.22	30.50		56	111	69	20	6	5	5	3	2	1	0	278
25.00	56.23	40.33		13	38	34	16	3	8	0	5	4	1	0	122
35.00	61.01	37.23		4	16	26	7	6	3	3	2	2	0	0	69
45.00	73.43	42.36		0	7	10	6	5	4	0	0	2	0	1	35
55.00	83.75	52.78		2	1	4	1	3	1	1	1	1	1	0	16
65.00	100.91	57.44		0	1	2	3	1	0	1	1	0	1	1	11
75.00	60.00	10.00		0	0	1	1	0	0	0	0	0	0	0	2
85.00	.00	.00		0	0	0	0	0	0	0	0	0	0	0	0
95.00	50.00	.00		0	0	1	0	0	0	0	0	0	0	0	1
TOTAL	39.62	33.11		325	377	242	74	34	24	11	13	12	5	2	1119

Table 1.2.1. Correlation between bending and springing stress level in the long run. From /14/.

A2 20

(ALL CONDITIONS)

A2.6 CORRELATION BETWEEN THE RICE DISTRIBUTION
PARAMETERS OF THE TOTAL STRESS

THE AMPLITUDES OR THE LOCAL MAXIMA OF THE TOTAL STRESS ARE SUPPOSED TO FOLLOW A RICE PROBABILITY FUNCTION.

THIS DISTRIBUTION FUNCTION HAS TWO PARAMETERS: THE ROOT-E VALUE DEFINED AS SQRT(2)*THE RMS-VALUE, AND THE SPECTRAL WIDTH EPSILON DEFINED AS SQRT(1 - (PEAK PERIOD/ZERO CROSSING PERIOD)**2).

IT WILL BE OF INTEREST TO SEE IF THESE PARAMETERS ARE STATISTICALLY INDEPENDENT OR IF THERE IS SOME CORRELATION.

IT WILL ALSO BE OF INTEREST TO KNOW THE CHARACTERISTIC VALUES OF THE SPECTRAL WIDTH, BECAUSE THE APPLICATION OF THE RAYLEIGH DISTRIBUTION COMMONLY USED, IS ONLY VALID FOR SMALL VALUES OF THE SPECTRAL WIDTH (LESS THAN EPSILON = 0.75).

THE INVESTIGATION IS BASED ON 1119 OBSERVATIONS.

SOME MAIN DATA OF THE OBSERVATIONS ARE LISTED IN THE FOLLOWING TABLE :

AVERAGE STRESS SPECTRAL WIDTH	8.6518835-01	DIMENSIONLESS
STANDARD DEVIATION OF STRESS SPECTRAL WIDTH	9.8364058-02	DIMENSIONLESS
LARGEST VALUE OF THE STRESS SPECTRAL WIDTH	9.9938352-01	DIMENSIONLESS
CORRESPONDING VALUE OF TOTAL STRESS ROOT-E VALUE	4.2000000+01	KP/CM**2
SMALLEST VALUE OF THE STRESS SPECTRAL WIDTH	3.8105959-01	DIMENSIONLESS
CORRESPONDING VALUE OF TOTAL STRESS ROOT-E VALUE	5.7000000+01	KP/CM**2
AVERAGE TOTAL STRESS ROOT-E VALUE	4.7243968+01	KP/CM**2
STANDARD DEVIATION OF TOTAL STRESS ROOT-E VALUE	3.2640409+01	KP/CM**2
LARGEST VALUE OF THE TOTAL STRESS ROOT-E VALUE	2.2300000+02	KP/CM**2
CORRESPONDING VALUE OF STRESS SPECTRAL WIDTH	8.9687238-01	DIMENSIONLESS
SMALLEST VALUE OF THE TOTAL STRESS ROOT-E VALUE	6.0000000+00	KP/CM**2
CORRESPONDING VALUE OF STRESS SPECTRAL WIDTH	9.6474199-01	DIMENSIONLESS

AN INTUITIVE IMPRESSION OF THE CORRELATION IS ALSO OBTAINED BY INSPECTION OF THE SCATTER DIAGRAM ON THE NEXT PAGE.

SCATTER DIAGRAM (ALL CONDITIONS)

THIS DIAGRAM SHOWS THE SIMULTANEOUS DISTRIBUTION OF OBSERVATIONS AND THE PARAMETERS OF THE CLASS-WISE AND MARGINAL DISTRIBUTIONS :

X: STRESS SPECTRAL WIDTH IN DIMENSIONLESS
Y: TOTAL STRESS ROOT-E VALUE IN KP/CM**2

CLASS MIDPOINT			X:	.35	.45	.55	.65	.75	.85	.95	TOTAL
	AVERAGE VALUE		Y:	50.00	52.50	43.33	48.49	52.46	55.74	38.65	47.52
		STANDARD DEVIATION	Y:	.00	25.37	19.84	22.60	25.90	38.87	31.67	33.57
Y:	X:	X:									
10.00	.92	.07		0	0	2	3	2	28	116	151
30.00	.87	.10		0	3	8	21	46	151	198	427
50.00	.85	.11		1	3	7	12	41	118	94	276
70.00	.83	.10		0	1	3	11	28	60	35	138
90.00	.83	.10		0	0	1	6	7	23	12	49
110.00	.84	.09		0	1	0	0	4	22	4	31
130.00	.87	.04		0	0	0	0	0	9	2	11
150.00	.86	.05		0	0	0	0	1	10	2	13
170.00	.90	.05		0	0	0	0	0	8	7	15
190.00	.88	.09		0	0	0	0	1	0	2	3
210.00	.90	.05		0	0	0	0	0	2	2	4
230.00	.85	.00		0	0	0	0	0	1	0	1
TOTAL	.86	.10		1	8	21	53	130	432	474	1119

Table 1.2.2. Correlation between stress level ($\sqrt{2}$ RMS) and the spectral width parameter ϵ. From /14/.

1.3 Conclusions.

If the attitude of a ship navigator is adopted, the problem can be stated as follows:
You know the springing and bending periods of the vessel. The first one is a farily constant one, while the bending period is roughly equal to the encountering wave period. You also know the RMS-value of the total stress which may be monitored in real time by the simplest type of a hull surveillance stress monitor. What you do not know is how the observed stress is shared between bending and springing. How important is it, for a safe operation of the vessel, to discern between bending and springing stress in this situation ?

The present work indicates the following main conclusions:

A To evaluate the largest stress Smax in a rough weather situation, there is no sense in discerning between springing and bending. The extreme load Smax may in any case be evaluated as

$$Smax = 1.4 \times RMS \times \sqrt{\ln N} \qquad (1.3.1)$$

where RMS is the value read from the monitor, and N is a rough mean value between the springing and bending number of cycles in the time period considered. The uncertainty introduced by rough estimate of N is completely exhausted by the natural, statistical dispersion of the largest stress, and both these sources of uncertainty are nearly exhausted by only a moderate measureing error of RMS. (Chapter 6 and 7).

B The relative uncertainty in the estimated largest value (1.3.1) due to natural dispersion under stationary conditions (that is relative dispersion of extreme value distribution, the correct values of RMS and N being exactly known) is with good approximation

$$\frac{\delta Smax}{Smax} = \frac{1}{2 \ln N} \qquad (1.3.2)$$

On the other hand the relative uncertainty in the extreme stress (1.3.1) due to uncertain monitoring of RMS is directly equal to the relative uncertainty in RMS itself.

C The fatigue rate increases when the springing share exceeds a certain limit. There is also an upper limit for the fatigue increase due to springing, and this limit is given by the bending-to-springing period ratio which may be of order 3-6. Thus, if the ship design has proved sensitive to fatigue damages, either by fatigue calculations at the design stage or by observation of cracks during the service stage, heavy springing should be avoided. (Chapter 8).

If the attitude of a ship designer is adopted, the problems will be somewhat different and may be stated as follows:

Bending and springing responses are regarded as two different phenomena. We have a number of hydrodynamic theories, methods and computer programs which may predict the bending stresses in the short and long term by a given service profile. We also have (presumably) theories, methods and computer programs for treating springing stresses in the same way.
The ship should, however, be designed to resist the <u>resulting</u> stresses, and the question is: How should bending and springing be combinded to give the resulting stress when those two components are known separately ?

The following conclusions may be of practical use:

D When the largest stress and the largest springing stress are separately known, a guiding estimate of the resulting maximum stress is

$$\text{Smax} \approx \sqrt{\text{Smax (bending)}^2 + \text{Smax (springing)}^2} \qquad (1.3.3)$$

The relation is assumed to hold in the short term as well as in the long term case. (Section 4.4)

E When the long term distribution of bending RMS and springing RMS are known (in terms of Weibull plots Fig.9.1.1) there exists an additive class of distributions which immediately indicates the long term distribution of the resulting RMS more or less roughly. In addition the long term distribution of the springing or bending share (springing resp. bending RMS relative to total RMS) is indicated. (Section 9.1).

F The spectral width ε entering the Rice distribution for local maxima is not a basic variable, but is uniquely determined by the springing-to-total RMS ratio (springing share x) and the bending-to-springing period ratio (τ). (Section 2.3).

G The limiting case of $\varepsilon=1$ can never be obtained in the present situation, while the opposite limit $\varepsilon=0$ is approached in both the pure springing and the pure bending case.
This indicates that a number of approximate formulae derived for $\varepsilon=0$ may be of rather general application. (Section 3.1).

H The Rice-distribution for positive maxima under stationary conditions may with good approximation be replaced by a generalized gamma distribution. The resemblance is exact in the limiting cases of $\varepsilon=0$ and $\varepsilon=1$. (Section 3.3).

I Several analytical probability distribution, exact and approximate, are available for the extreme stress under stationary conditions by known zero crossing period. (Chapter 5).

J Probability distribution for the extreme stress by unknown period is derived in analytical form, but this distribution does not deviate significantly from the distribution with period fixed at the mean value between bending and springing. (Chapter 6).

K For given long term probability distribution of RMS, an analytical procedure is suggested which attaches a generalized gamma function to the long term distribution of stress peaks. (Chapter 10).

2. GENERAL PROPERTIES OF A TWO-COMPONENT STRESS SPECTRUM

2.1 Basic variables

The resultant stress presently considered consists of two distinguished spectral components which will be termed the bending stress due to quasistatic wave action, and the springing stress due to resonans vibration. Regarded separately, each stress component is narrow-banded, and a short term stationary stress state is completely characterized by the four variables:

σ_B Root Mean Square (RMS) of the bending stress
T_B Average period of the bending stress
σ_S RMS of the springing stress
T_S Average period of the springing stress

If the power spectral density of the total stress is termed $S(\omega)$, a function of the circular frequency ω, the spectral moment of order n can in general be written:

$$M_n = \int_0^\infty \omega^n S(\omega)\,d\omega = \left(\frac{2\pi}{T_B}\right)^n \sigma_B{}^2 + \left(\frac{2\pi}{T_S}\right)^n \sigma_T{}^2 \qquad (2.1.1)$$

Based on the spectral moments of order 0, 2 and 4 the following three parameters of the total stress may be defined:

- RMS-value σ (or variance σ^2)

$$\sigma = \sqrt{M_0} = \sqrt{\sigma_B{}^2 + \sigma_S{}^2} \qquad (2.1.2)$$

- Average zero-crossing period T_z

$$T_z = 2\pi\sqrt{\frac{M_0}{M_2}} = \sqrt{\frac{\sigma_B{}^2 + \sigma_S{}^2}{\sigma_B{}^2/T_B{}^4 + \sigma_S{}^2/T_S{}^4}} \qquad (2.1.3)$$

- Average peak period T_P

$$T_P = 2\pi\sqrt{\frac{M_2}{M_4}} = \sqrt{\frac{\sigma_B{}^2/T_B{}^2 + \sigma_S{}^2/T_S{}^2}{\sigma_B{}^2/T_B{}^4 + \sigma_S{}^2/T_S{}^4}} \qquad (2.1.4)$$

From these basic variables one may derive a number of related parameters which do not convey new information, but which have significance for physical or historical reasons:

- The ratio α between peak- and zero-crossing period

$$\alpha = \frac{T_P}{T_Z} = \frac{\sigma_B^2/T_B^2 + \sigma_S^2/T_S^2}{\sqrt{(\sigma_B^2/T_B^4 + \sigma_S^2/T_S^4)(\sigma_B^2 + \sigma_S^2)}} = \sqrt{1 - \varepsilon^2} = 2a - 1 \tag{2.1.5}$$

- The spectral width ε

$$\varepsilon = \sqrt{1 - \alpha^2} = 2\sqrt{a(1 - a)} \tag{2.1.6}$$

- The fraction of positive maxima a

$$a = \tfrac{1}{2}(1 + \sqrt{1 - \varepsilon^2}) = \tfrac{1}{2}(1 + \alpha) \tag{2.1.7}$$

The spectral width ε has been the prevailing shape parameter of the Rice distribution in the literature /1/, /2/. Some authors have preferred the period ratio α , for example /3/, /4/. The related parameter a is essential by considering the distribution of positive peaks only, see /5/, and appears as the shape parameter in the generalized gamma distribution approximated to the Rice distribution, see Section 3.3.

2.2. Dimensionless Representation

It is convenient to perform the derivations within a dimensionless system of variables.
As time unit is chosen the springing period T_S which is the most stable of the time variables introduced. Periods measured with T_S as unit will be denoted τ with a corresponding subscript.

As stress unit in the short term case will be chosen the total RMS value, σ. This is the quantity which is for instance most easy to monitor in service. Stress variables measured with σ as unit will be denoted x with a corresponding subscript.

τ written without subscript denotes the dimensionless <u>bending</u> period.
x written without subscript denotes the dimensionless <u>springing</u> RMS.
Hence the variables previously defined may be re-written as follows:

- RMS of bending stress x_B, bending share

$$x_B = \frac{\sigma_B}{\sigma} = \frac{\sigma_B}{\sqrt{\sigma_B^2 + \sigma_S^2}} = \sqrt{1 - x_S^2} = \sqrt{1 - x^2} \tag{2.2.1}$$

- Period of bending stress τ_B

$$\tau_B = \tau = \frac{T_B}{T_S} \tag{2.2.2}$$

- RMS of springing stress X, springing share

$$x_S = x = \frac{\sigma_S}{\sigma} = \frac{\sigma_S}{\sqrt{\sigma_S^2 + \sigma_B^2}} \tag{2.2.3}$$

- Average zero-crossing period τ_Z

$$\tau_Z = \frac{T_Z}{T_S} = \frac{1}{\sqrt{x^2 + x_B^2/\tau^2}} = \frac{\tau}{\sqrt{1 + x^2(\tau^2 - 1)}} \tag{2.2.4}$$

- Average peak period τ_P

$$\tau_P = \frac{T_P}{T_S} = \sqrt{\frac{x_B^2/\tau^2 + x^2}{x_B^2/\tau^4 + x^2}} = \tau\sqrt{\frac{1 + x^2(\tau^2 - 1)}{1 + x^2(\tau^4 - 1)}} \tag{2.2.5}$$

- Ratio between peak- and zero-crossing period

$$\alpha = \frac{\tau_P}{\tau_Z} = \frac{X_B{}^2 + X^2\tau^2}{\sqrt{X_B{}^2 + X^2\tau^4}} = \frac{1 + X^2(\tau^2 - 1)}{\sqrt{1 + X^2(\tau^4 - 1)}} = 2a - 1 \qquad (2.2.6)$$

- Spectral width ε

$$\varepsilon = \sqrt{1 - \alpha^2} = \left[1 - \frac{[1 + X^2(\tau^2 - 1)]^2}{1 + X^2(\tau^4 - 1)}\right]^{\frac{1}{2}} = \left[\frac{X^2(1 - X^2)(\tau^2 - 1)}{1 + X^2(\tau^4 - 1)}\right]^{\frac{1}{2}}$$

(2.2.7)

- Fraction of positive maxima a

$$a = \tfrac{1}{2}\left[1 + \frac{1 + X^2(\tau^2 - 1)}{\sqrt{1 + X^2(\tau^4 - 1)}}\right] = \tfrac{1}{2}\left[1 + \sqrt{1 - \varepsilon^2}\right] = \tfrac{1}{2}[1 + \alpha]$$

(2.2.8)

2.3 Considerations of the Spectral Width

As observed from equation (2.2.7), the spectral width parameter ε is completely determined by the springing RMS-to-total RMS ratio x, and the bending-to-springing period ratio τ.
The equation is

$$\varepsilon^2 = \frac{X^2(1 - X^2)(\tau^2 - 1)^2}{1 + X^2(\tau^2 - 1)(\tau^2 + 1)} \tag{2.3.1}$$

From this we may conclude:

- $\varepsilon = 0$ for $x = 0$. This occurs when $\sigma_S = 0$ and the stress is pure bending.
- $\varepsilon = 0$ for $(1 - X^2) = \sigma_B{}^2/\sigma^2 = 0$. That is: there is no bending, and the stress is pure springing.
- $\varepsilon = 0$ for $\tau = T_B/T_S = 1$. That is: the spectral peaks due to springing and bending coincide, and there is in reality only one peak in the stress spectrum.
- Differentiation with respect to X keeping τ constant reveals that ε has its maximum value when

  $$\frac{\sqrt{1-X^2}}{X} = \tau \quad \text{that is} \quad \frac{\sigma_B}{\sigma_S} = \frac{T_B}{T_S} \tag{2.3.2}$$

  The spectral width is then given by

  $$\varepsilon_{max} = \frac{\tau^2 - 1}{\tau^2 + 1} = \frac{T_B{}^2 - T_S{}^2}{T_B{}^2 + T_S{}^2} = 1 - \frac{2}{\tau^2 + 1} \tag{2.3.3}$$

  or by

  $$\varepsilon_{max} = 1 - 2X^2 = 1 - 2(\sigma_S/\sigma)^2 \tag{2.3.4}$$

- $\varepsilon = 1$ is obtained only when $\tau \to \infty$ and $X \to 0$, that is $T_S >> T_B$ and $\sigma_S >> \sigma_B$

The spectral width ε is shown as a function of the springing share $X = \sigma_S/\sigma$ for selected values of the relative bending period $\tau = T_B/T$ in Fig. 2.3.1.

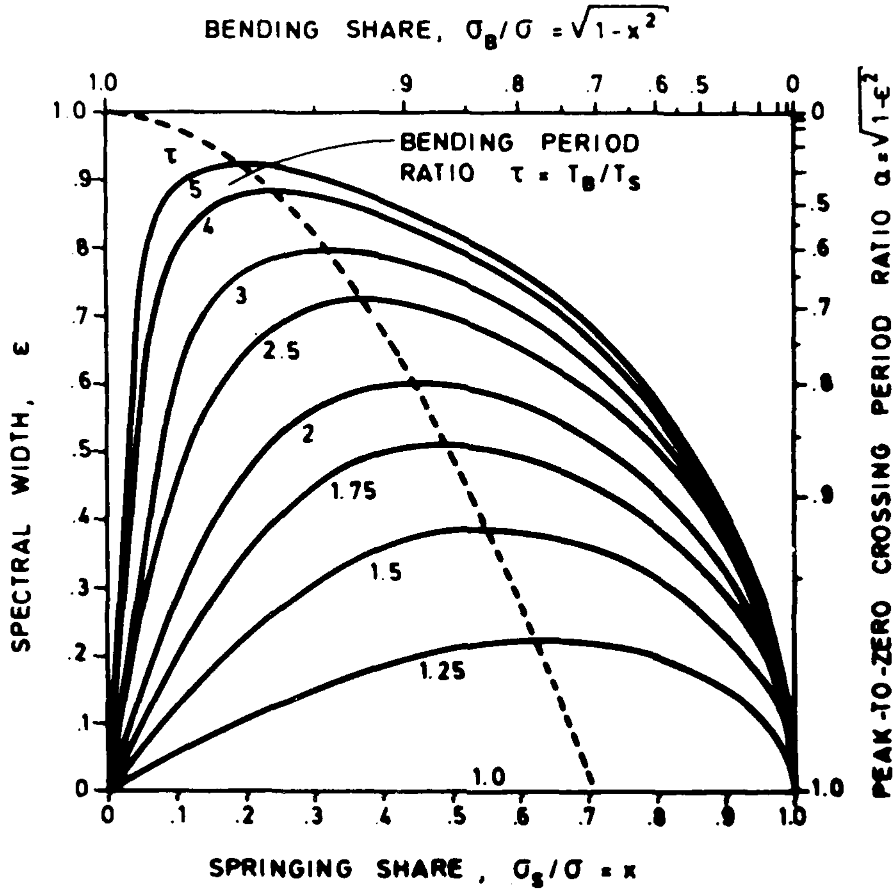

Fig. 2.3.1. Spectral width ε as a function of the springing share $X = \sigma_S/\sigma$ for selected values of the bending-to-springing period ratio $\tau = T_B/T_S$.

This figure shows in more detail how the spectral width becomes zero when the stress is either pure bending, $\sigma_S/\sigma = 0$, or pure springing, $\sigma_S/\sigma = 1.0$. And also how the spectral width disappears when the springing and bending periods approaches each other at $\tau = 1$.

The maximum values of the spectral width are located on the dashed line which is the parabola described by (2.3.4).

The value $\varepsilon = 1$ is an ideal case which is never realized in practice.

2.4 Consideration of Periods

In a given time interval t, the number of local maxima is N_p

$$N_p = \frac{t}{T_p} = \frac{t}{T_S}\sqrt{\frac{1+x^2(\tau^4-1)}{[1+x^2(\tau^2-1)]\tau^2}} = \frac{t}{T_B}\sqrt{\frac{1+x^2(\tau^4-1)}{1+x^2(\tau^2-1)}} \qquad (2.4.1)$$

The peak period T_p is given in (2.1.4).
The dimensionless representation is given in (2.2.5).
The term t/T_S on the right side is the number of springing cycles in the time t considered. The square root expression is therefore the ratio between local maxima and number of springing cycles experienced within an arbitrary time interval, and is graphed in Fig. 2.4.1.

It is observed that the peak period very rapidly becomes equal to the springing period, more rapidly the longer the bending period is.

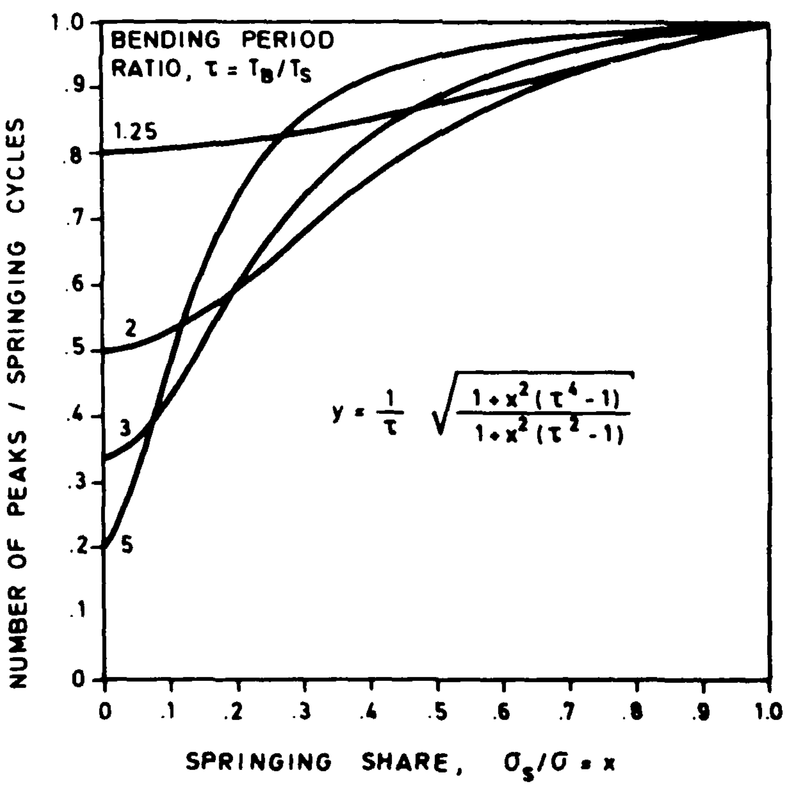

Fig. 2.4.1 Number of local maxima within a time interval, as function of springing cycles.

Similarly, in a given interval t, the number of zero crossings N_Z is

$$N_Z = \frac{t}{T_Z} = \frac{t}{T_S}\,\frac{1}{\tau}\sqrt{1+x^2(\tau^2-1)} = \frac{t}{T_B}\sqrt{1+x^2(\tau^2-1)} \qquad (2.4.2)$$

T_Z is the zero crossing period given in (2.1.3), dimensionless in (2.2.4). As t/T_S is the number of springing periods within t, the square root term is the ratio between zero crossing periods and springing periods in any arbitrary time interval. The ratio is graphed in Fig. 2.4.2.

Fig. 2.4.1 and Fig. 2.4.2 show how, depending on the circumstances, the peak as well as the zero-crossing period will always be situated somewhere between the bending and springing periods.

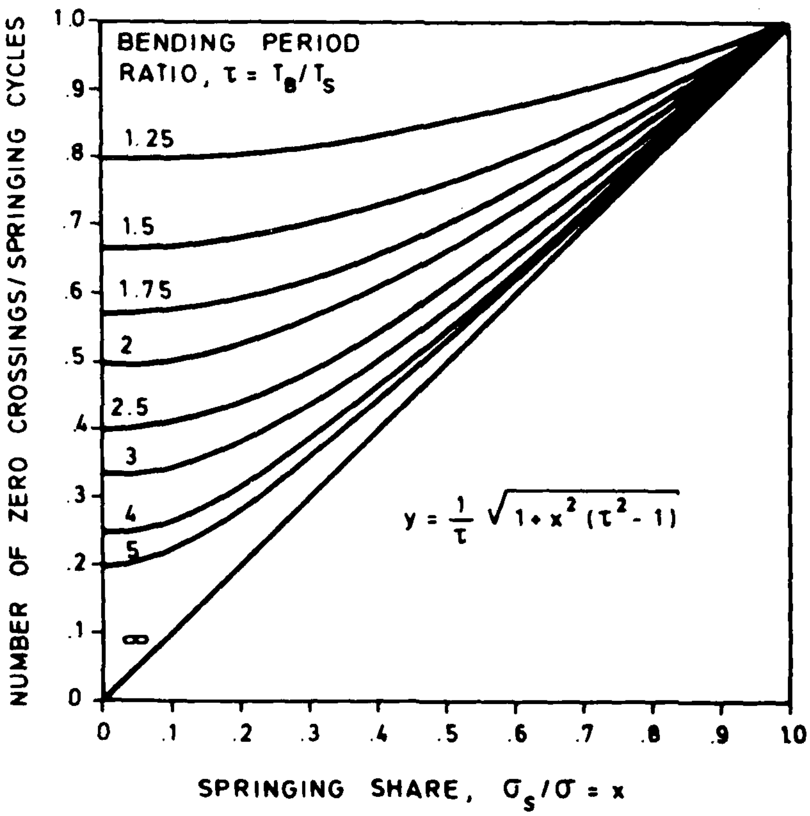

Fig. 2.4.2 Number of zero crossing periods within a time interval as a fraction of springing periods.

The fraction of positive maxima is given by the parameter a in (2.1.7) and (2.2.8). Hence the number of positive maxima, denoted N_p^+, in the time t is

$$N_p^+ = aN_p = \tfrac{1}{2}[1+\sqrt{1-\varepsilon^2}]N_p = \frac{N_p+N_z}{2} \qquad (2.4.3)$$

that is, the mean value between the number of peaks and zero-up-crossings. The average period between positive peaks, denoted T_p^+ is hence

$$T_p^+ = \frac{2}{1/T_p+1/T_z} = \frac{2T_pT_z}{T_p+T_z} = \frac{T_p}{a} \qquad (2.4.4)$$

Correspondingly the fraction of negative maxima is (1-a), which gives a number of negative maxima denoted N_p^- of

$$N_p^- = (1-a)N_p = \tfrac{1}{2}[1-\sqrt{1-\varepsilon^2}]N_p = \frac{N_p-N_z}{2} \qquad (2.4.5)$$

The average period between the negative maxima, denoted T_p^-, is

$$T_p^- = \frac{2}{1/T_p-1/T_z} = \frac{2T_zT_p}{T_z-T_p} \qquad (2.4.6)$$

These expressions will be of importance later.

The fraction of positive maxima is uniquely determined by the bending-to-springing period ratio τ and the springing share x as it appears from equation (2.2.8).
The dependence is graphed in Fig. 2.4.3.

Fraction of positive maxima is essential in the probability distribution of positive maxima, Section 3.2 and 4.5 and also for the approximation with generalized gamma distributions, Section 3.3 and 4.6.

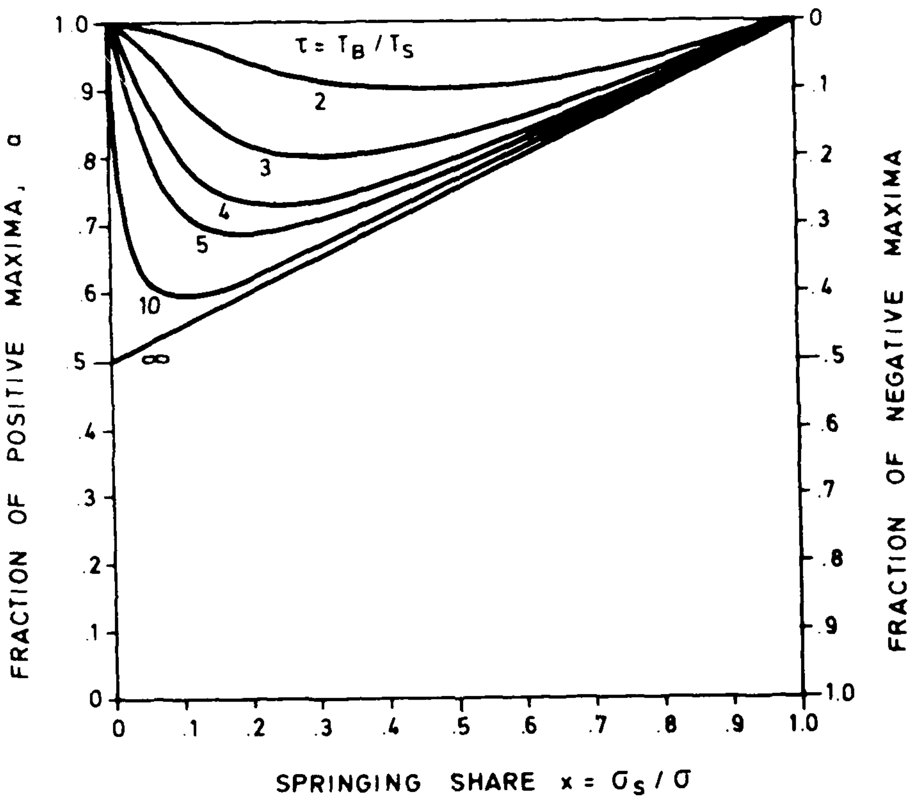

Fig. 4.2.3 Fraction of positive maxima as a function of the period ratio τ and the springing share x. (Equation (3.8)).

3. DISTRIBUTION OF MAXIMA UNDER STATIONARY CONDITIONS

We consider a time interval during which the sea conditions and the ship speed and course do not change significantly. In the real life this will only be the case in short term intervals of a couple of hours duration. However, consideration of such stationary intervals of, say, 20 years duration is also of interest because some long term trends may be qualitatively indicated.

Looked separately, the bending and springing stresses are assumed to be gaussian random processes with RMS values σ_B and σ_S respectively. That is: If the bending stress component is filtered out and sampled, the sample values over a sufficiently long period will conform with a normal probability law with standard deviation σ_B. Similarly sampled data from the springing stress component will conform with a normal probability law with standard deviation σ_S. If the two spectral components are statistically independent, sampled data of the complete stress signal conform with a normal distribution with standard deviation σ, equal to the RMS-value.

Looked separately, the bending and springing stresses are also assumed to be narrow-banded. That means, among other things, that they appear as a sequence of slowly modulated amplitudes which conform with a Rayleigh probability distribution with parameter $\sqrt{2}\sigma_B$ for the bending stress and $\sqrt{2}\sigma_S$ for the springing stress component. The distribution of amplitudes coincides with the sampled value distribution of the envelope of the respective components. In the narrow banded case the stress peaks, or local maxima, coincide with the amplitudes, and represent a sampling of the stress envelope with a sampling period, equal to the period of the respective stress components. Since there is one zero-up crossing and one positive peak in each cycle, there is little or no difference between the zero crossing period and the peak for a narrow-banded stress component.

When the stress components are superposed upon each other, the concepts of "envelope" and "amplitude" loose the significance. What is still significant is the sequence of peaks, or local

maxima. This makes little conceptual difficulties for the extreme stress prediction, since the extreme stress within a certain time interval is easily definable as the largest local maximum value. In fatigue, however, the complications become considerable because the amplitude-concept is lost. By counting cycles for fatigue life prediction, however, one can evidently not identify the stress cycles with the peaks.

Alternative parameters to the RMS-value σ are:

- The "ROOT-E" value $\sqrt{E} = \sqrt{2}\sigma$ which is the Rayleigh distribution parameter for single amplitudes.

- The significant height, or double amplitude, $H_s=4\sigma$ which is preferred in the analogy of wave motion.

3.1 Exact distribution of local maxima

For a broad-band stress history, which also covers the present two-component case, the peaks or local maxima conform with a Rice probability law /1/ /2/ which has two parameters:

- the RMS-value σ
- the spectral width ε (or any related parameter α or a).

Denote the sequence of N stress peaks

$$S_1 \; S_2 \; S_3 \; \ldots\ldots S_N. \tag{3.1.1}$$

Normalize the stress peaks with respect to the RMS-value σ and define the sequence of dimensionless peaks

$$Z_1 \; Z_2 \; Z_3 \; \ldots\ldots Z_N, \qquad Z_m = S_m/\sigma \tag{3.1.2}$$

The probability density function of the stress peaks is

$$g(Z) = \frac{1}{\sqrt{2\pi}} \; e^{-\frac{Z^2}{2\varepsilon^2}} + \sqrt{1-\varepsilon^2} \; \phi\left(\frac{\sqrt{1-\varepsilon^2}}{\varepsilon} Z\right) Z \; e^{-\frac{Z^2}{2}} \tag{3.1.3}$$

$\phi(\;)$ is the normal probability integral

$$\phi(x) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{X} e^{-\frac{t^2}{2}} \, dt \tag{3.1.4}$$

Special cases of (3.1.3) are:

$\varepsilon = 0$	$g(Z) = Z \; e^{-Z^2/2}$	(Rayleigh)	(3.1.5)
$\varepsilon \ll 1 \ll Z$	$g(Z) = \sqrt{1-\varepsilon^2} \; Z \; e^{-Z^2/2}$		(3.1.6)
$\varepsilon = 1$	$g(Z) = \frac{1}{\sqrt{2\pi}} \; e^{-Z^2/2}$	(Normal)	(3.1.7)
$Z = 0$	$g(0) = \varepsilon/\sqrt{2\pi}$		(3.1.8)

The probability of exceedance is

$$Q(Z) = 1-\phi(Z/\varepsilon) + \sqrt{1-\varepsilon^2}\ \phi\left(\frac{\sqrt{1-\varepsilon^2}}{\varepsilon}Z\right)e^{-Z^2/2} \qquad (3.1.9)$$

with the special cases:

$$\varepsilon = 0 \qquad Q(Z) = e^{-Z^2/2} \qquad (3.1.10)$$

$$\varepsilon << 1 << Z \qquad Q(Z) = \sqrt{1-\varepsilon^2}\ e^{-Z^2/2} \qquad (3.1.11)$$

$$\varepsilon = 1 \qquad Q(Z) = 1-\phi(Z) \qquad (3.1.12)$$

$$Z = 0 \qquad Q(0) = \tfrac{1}{2}(1+\sqrt{1-\varepsilon^2}) = a \qquad (3.1.13)$$

The cumulative probability function P(Z) appears immediately from the exceedance probability through

$$P(Z) = 1 - Q(Z) \qquad (3.1.14)$$

The moments of the distribution are:

$$\text{Mean value:} \qquad \mu_1 = \bar{Z} = \sqrt{\frac{\pi}{2}}\ \sqrt{1-\varepsilon^2} \qquad (3.1.15)$$

$$\text{Variance :} \qquad \mu_2 = \overline{(Z-\bar{Z})^2} = 1-(\frac{\pi}{2}-1)(1-\varepsilon^2) \qquad (3.1.16)$$

$$\text{3rd central moment:} \ \mu_3 = \overline{(Z-\bar{Z})^3} = \sqrt{\frac{\pi}{2}}\ (\pi-3)(1-\varepsilon^2)^{3/2} \qquad (3.1.17)$$

Hence follow the dimensionless coefficients:

$$\text{Coefficient of variation} \quad \lambda = \mu_2^{\frac{1}{2}}/\mu_1 = \sqrt{\frac{2}{\pi}}\ \sqrt{1-\frac{\pi}{2}+\frac{1}{1-\varepsilon^2}} \qquad (3.1.18)$$

$$\text{Coefficient of skewness} \quad \beta = \mu_3/\mu_2^{\frac{3}{2}} = \sqrt{\frac{\pi}{2}}(\pi-3)\left\{\frac{1-\varepsilon^2}{1-(\pi/2-1)(1-\varepsilon^2)}\right\}^{3/2} \qquad (3.1.19)$$

A graph of the exceedance probability is given in Fig. 3.1.1. The limiting cases of $\varepsilon = 0$ and $\varepsilon = 1$ are plotted together with $\varepsilon = 0.5$, 0.9 and 0.95. With the extreme value prediction in mind,

the high-peak-low-probability region is of most interest. Hence, looking to the region about $Z = 4$, $Q = 10^{-4}$, it is observed that Q is reduced by roughly one decade when ε goes from 0 to 1 under constant σ. Half a decade, however, is occupied by the transition from $\varepsilon = 0$ to $\varepsilon = 0.95$.

Keeping the probability level constant, it is observed that the argument Z is reduced by about 0.6, or 15%, when ε goes from 0 to 1. An amount of 0.3, or 7%, however, is occupied by the transition from 0 to 0.95.

These observations indicate that the special cases of low ε and large stress, equations (3.1.6) and (3.1.11), may be valid for fairly large ε-values.

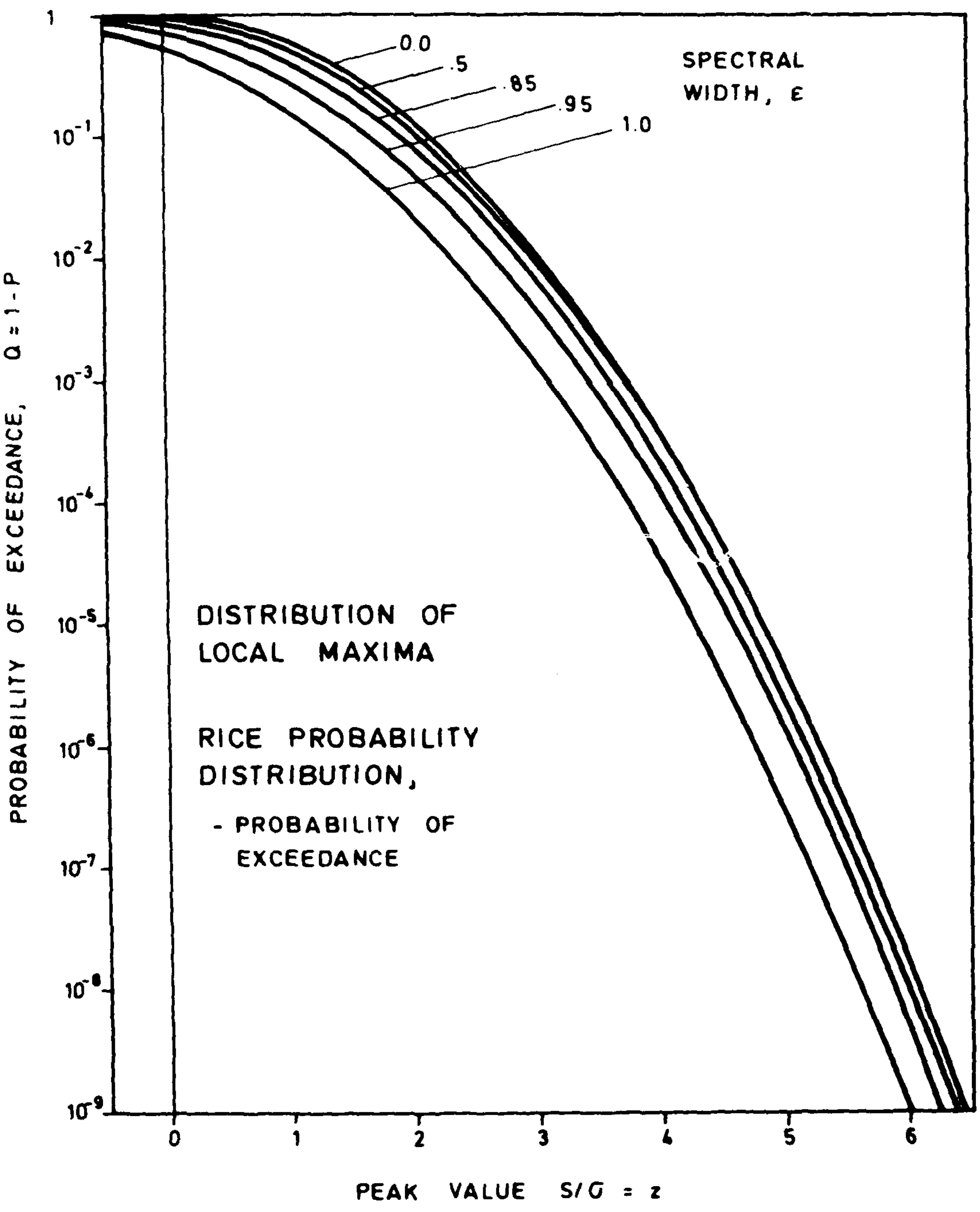

Fig. 3.1.1 Graph of the probability of exceedance of the Rice probability distribution.

3.2 Distribution of positive maxima

For $\varepsilon > 0$ there is a certain fraction of negative maxima and an equal fraction of positive minima, which is given by (3.1.13). The number of positive maxima is given in (2.4.3), and is

$$N_p^+ = Q(0)N_p = aN_p = \frac{N_p+N_z}{2} = \frac{\sqrt{1+x^2(\tau^4-1)}+\tau[1+x^2(\tau^2-1)]}{2\tau\sqrt{1+x^2(\tau^2-1)}}\frac{t}{T_s} \tag{3.2.1}$$

The probability distribution of the positive maxima is found by truncating the ordinary Rice distribution at Z = 0. This gives the probability density function, corresponding to (8.3)

$$g^+(Z) = \frac{g(z)}{Q(0)} = \frac{1}{\sqrt{2\pi}}\frac{\varepsilon}{a}e^{-\frac{Z^2}{2\varepsilon^2}} + \frac{\sqrt{1-\varepsilon^2}}{a}\phi(\frac{\sqrt{1-\varepsilon^2}}{\varepsilon}Z)Ze^{-\frac{Z^2}{2}} \tag{3.2.2}$$

where

$$a = \tfrac{1}{2}[1+\sqrt{1-\varepsilon^2}] = Q(0) \tag{3.2.3}$$

The probability of exceedance, corresponding to (8.9) is

$$Q^+(Z) = \frac{Q(Z)}{Q(0)} = \frac{1}{a}[1-\phi(Z/\varepsilon)] + \frac{\sqrt{1-\varepsilon^2}}{a}\phi(\frac{\sqrt{1-\varepsilon^2}}{\varepsilon}Z)e^{-Z^2/2} \tag{3.2.4}$$

with the narrow band approximation corresponding to (3.1.9)

$$\varepsilon << 1 << Z: \qquad Q^+(Z) = \frac{\sqrt{1-\varepsilon^2}}{a}e^{-Z^2/2} \tag{3.2.5}$$

The exceedance probability is graphed in Fig. 3.2.1.
The cumulative probability function is given by

$$P^+(Z) = 1 - Q^+(Z) = \frac{P(Z) - P(0)}{Q(0)} \tag{3.2.6}$$

Extreme values have been studied in terms of the truncated distribution by Ochi /5/.
The truncated distribution will be applied later in the approximation with generalized gamma distribution, Section 3.3, and in discussion of extreme value, Section 4.5.

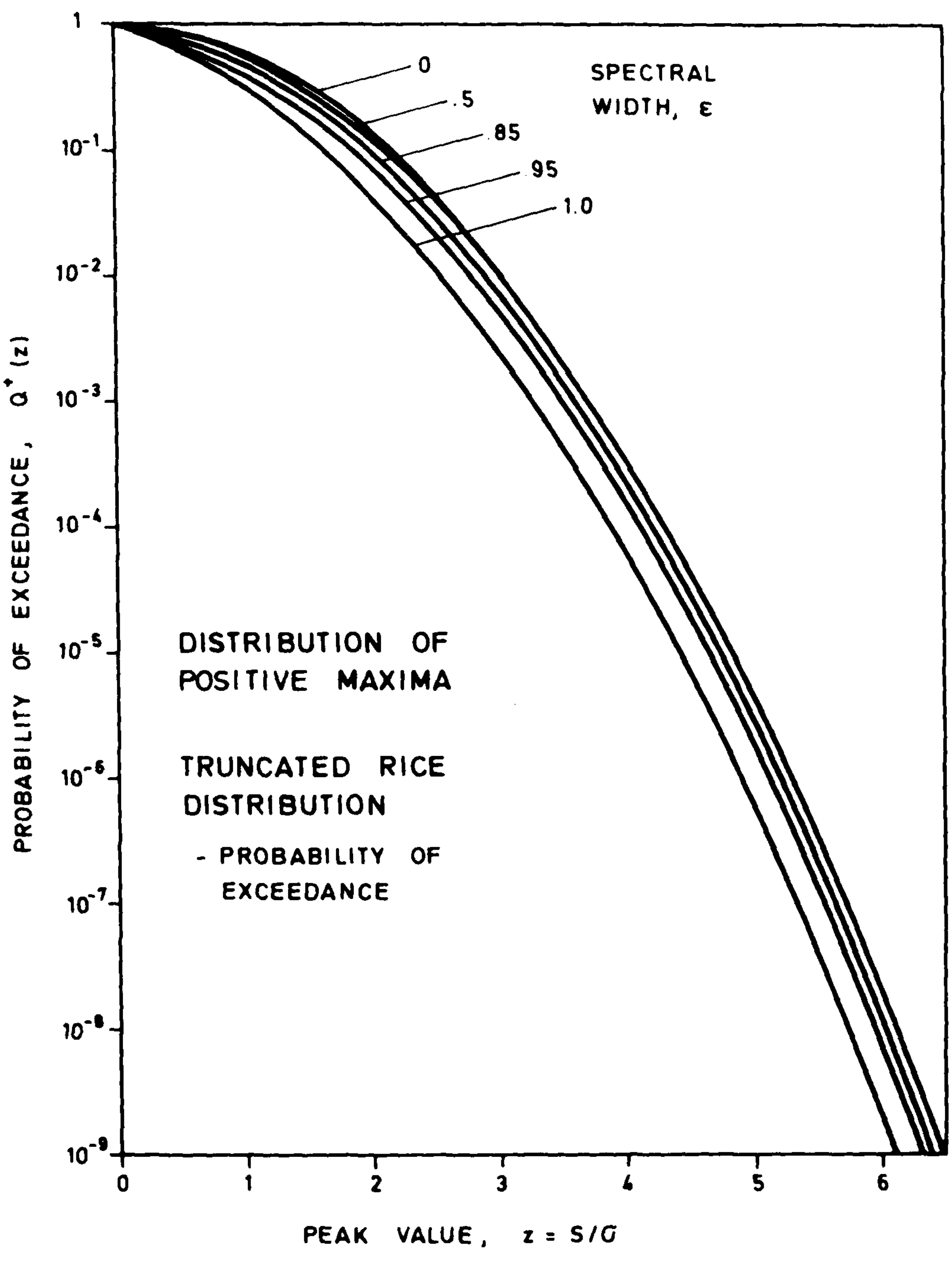

Fig. 3.2.1 Graph of the probability of exceedance for the truncated Rice distribution for positive stress peaks, or local maxima.

3.3 Approximation with generalized gamma distribution

The generalized gamma distribution has the density function

$$f(Z) = \frac{h}{\Gamma(a)A}\left(\frac{Z}{A}\right)^{ah-1} e^{-(Z/A)^h} \tag{3.3.1}$$

where a, h and A are parameters. This distribution covers a lot of well-known distributions as special cases, see Table 3.3.1.

With reference to Article 9 it is observed that the limiting case of the truncated Rice distribution with $\varepsilon = 0$, that is the Rayleigh distribution, is equal to the generalized gamma distribution with

$$\begin{aligned} a &= 1 \\ h &= 2 \\ A &= \sqrt{2} \end{aligned} \tag{3.3.2}$$

Similarly, the broad band limiting case $\varepsilon = 1$, that is the

a	h	A	Distribution function
1/2	2	$\sqrt{2}\sigma$	One sided normal distribution
1/2	2	σ	Error function
a	1	2	Elementary gamma distribution
n/2	1	2	χ^2-distribution with n degrees of freedom
1	2	$\sqrt{2}\sigma$	Rayleigh distribution
3/2	2	$\sqrt{2}\sigma$	Maxwell distribution
1	1	A	Exponential distribution
1	$h>0$	A	Two parameter Weibull distribution
1	$h<0$	A	Fréchet distribution
1	$\frac{\bar{x}}{\sigma}>>1$	$\bar{x}$	Approximate normal distribution with expectation $\bar{x}$ and standard deviation σ
1	∞	A	δ-distribution. Constant x = A

Table 3.3.1 Special cases of the generalized gamma distribution.

one sided normal distribution, coincides with the generalized gamma distribution with parameters

$$a = 1/2$$
$$h = 2 \qquad (3.3.3)$$
$$A = \sqrt{2}$$

It is hence reasonable to assume that the intermediate family of truncated Rice distributions with arbitrary ε can be approximated with generalized gamma distributions with parameters

$$a = \tfrac{1}{2}[1-\sqrt{1+\varepsilon^2}]$$
$$h = 2 \qquad (3.3.4)$$
$$A = \sqrt{2}$$

This can also be shown to be the case.
The distribution of positive maxima has hence approximately the probability density function

$$g^+(Z) = \frac{\sqrt{2}}{\Gamma(a)} \left(\frac{Z}{\sqrt{2}}\right)^{2a-1} e^{-Z^2/2} \qquad (3.3.5)$$

and the exceedance probability function

$$Q^+(Z) = \Gamma(a;\ Z^2/2)/\Gamma(a) \qquad (3.3.6)$$

where the complete and incomplete gamma functions are defined by

$$\Gamma(a;x) = \int_x^\infty t^{a-1}\, e^{-t} dt \quad , \quad \Gamma(a) = \Gamma(a;0) \qquad (3.3.7)$$

Graph of the exceedance probability is given in Fig. 3.3.1.

Further information about the generalized gamma distribution is given in /6/ or /7/ among others.

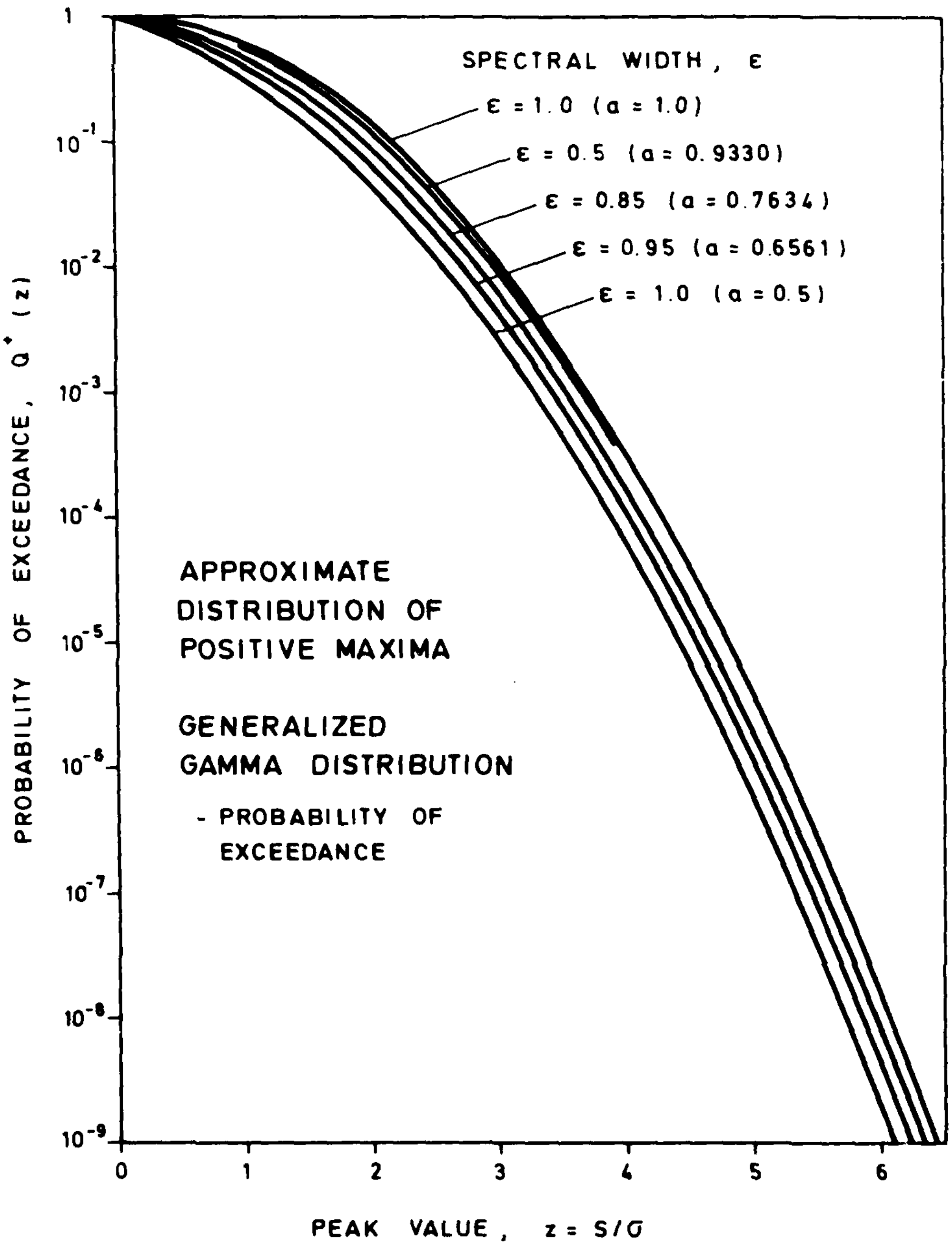

Fig. 3.3.1 Graph of the probability of exceedance approximated by the generalized gamma distribution.

4. CHARACTERISTIC EXTREME VALUE

4.1 Elementary considerations

Consider a time interval t under stationary conditions with a given RMS value σ of the complete stress signal. One then has the simple and well known estimate for the extreme value

$$S_c = \sigma\sqrt{2 \ln N} \tag{4.1.1}$$

which is obtained by putting the exceedance value Q equal to 1/N.

This formula is correct only when $\varepsilon = 0$, and from the considerations in Section 2.3 this takes place in the two limiting cases when the springing share $x = \sigma_S/\sigma = 0$ and 1.

In the case $x = 0$, the stress is a pure bending process, and the number of cycles is

$$N_B = \frac{t}{T_B} \tag{4.1.2}$$

In the opposite case $x = 1$, the stress is a pure springing process, and the number of cycles is

$$N_S = \frac{t}{T_S} = \frac{t}{T_B} \cdot \tau = N_B \cdot \tau \qquad (\tau = T_B/T_S) \tag{4.1.3}$$

Thus, when the stress goes from pure bending to pure springing with the same RMS, the extreme value increases with a factor

$$\frac{S_c \text{ (pure springing)}}{S_c \text{ (pure bending)}} = \sqrt{\frac{\ln N_S}{\ln N_B}} = \sqrt{1 + \frac{\ln \tau}{\ln N_B}} \tag{4.1.4}$$

Results are given in Table 4.1.1 in terms of *percentage increase* in extreme value when stress goes from pure bending to pure springing. One has reason to believe that this is a maximum increase, and that the increase in a mixture of bending and springing lies somewhere between.

One may conclude from these results that, when the total stress RMS is given, the presence of springing can at most elevate the extreme stress with about 10-15% in the short time case. For the long term case the table indicate an increase of order 5%.

τ	Number of bending periods in the time interval, N_B						
	10^2	10^3	10^4	10^5	10^6	10^7	10^8
2	7.26%	4.90%	3.69%	2.97%	2.48%	2.13%	1.86%
3	11.3	7.66	5.80	4.66	3.90	3.35	2.94
4	14.1	9.58	7.26	5.85	4.90	4.21	3.69
5	16.2	11.0	8.39	6.76	5.66	4.87	4.28
6	18.0	12.2	9.29	7.50	6.29	5.41	4.75
7	19.3	13.2	10.1	8.12	6.81	5.86	5.15
8	20.5	14.1	10.7	8.66	7.26	6.26	5.49

Table 4.1.1 Percentage increase in extreme value when the stress goes from pure bending to pure springing under constant RMS.

4.2 Characteristic extreme at arbitrary springing share

One considers a time interval t which contains a sequence of N_p stress peaks in the total stress history. A characteristic value for the extreme, or maximum stress peak is obtained by putting $Q(Z) = 1/N_p$ in equation (3.1.11). This gives:

$$S_c = \sigma\sqrt{2 \ln\sqrt{1-\varepsilon^2}\, N_p} = \sigma\sqrt{2 \ln (T_p/T_z)(t/T_p)} = \sigma\sqrt{2 \ln N_z} \qquad (4.2.1)$$

where N_z is the number of zero crossings in the same time interval.
Equation (4.2.1) is valid for small and moderate values of ε only. But since ε is zero in both the limiting cases: springing is all or none, and since ε can never become = 1.0, it is reasonable to believe (4.2.1) to be generally applicable in the present problem.

Equation (4.2.1) will then replace equation (4.1.1) which was only valid for $\varepsilon = 0$. Thus introducing the number of zero crossings from (2.1.2) into (11.1), one may derive the ratio between the maximum stress at arbitrary springing share and the maximum stress in the pure bending case:

$$\frac{S_c \text{ (arbitrary springing)}}{S_c \text{ (pure bending)}} = \sqrt{1 + \frac{1}{2}\, \frac{\ln[1+x^2(\tau^2-1)]}{\ln N_B}} \qquad (4.2.2)$$

This is a generalization of (4.1.4), the latter giving the limiting case of $x = 1$ only.

Some results are shown in Fig. 4.2.1, for some selected values of the bending-to-springing period ratio τ and the time intervals given in terms of bending cycles. This figure shows the transition of the extreme stress amplitude by increasing springing share up to the pure springing case. And with reference to the Table 4.1. it may be concluded that the extreme stress increases roughly linearly with the springing share.

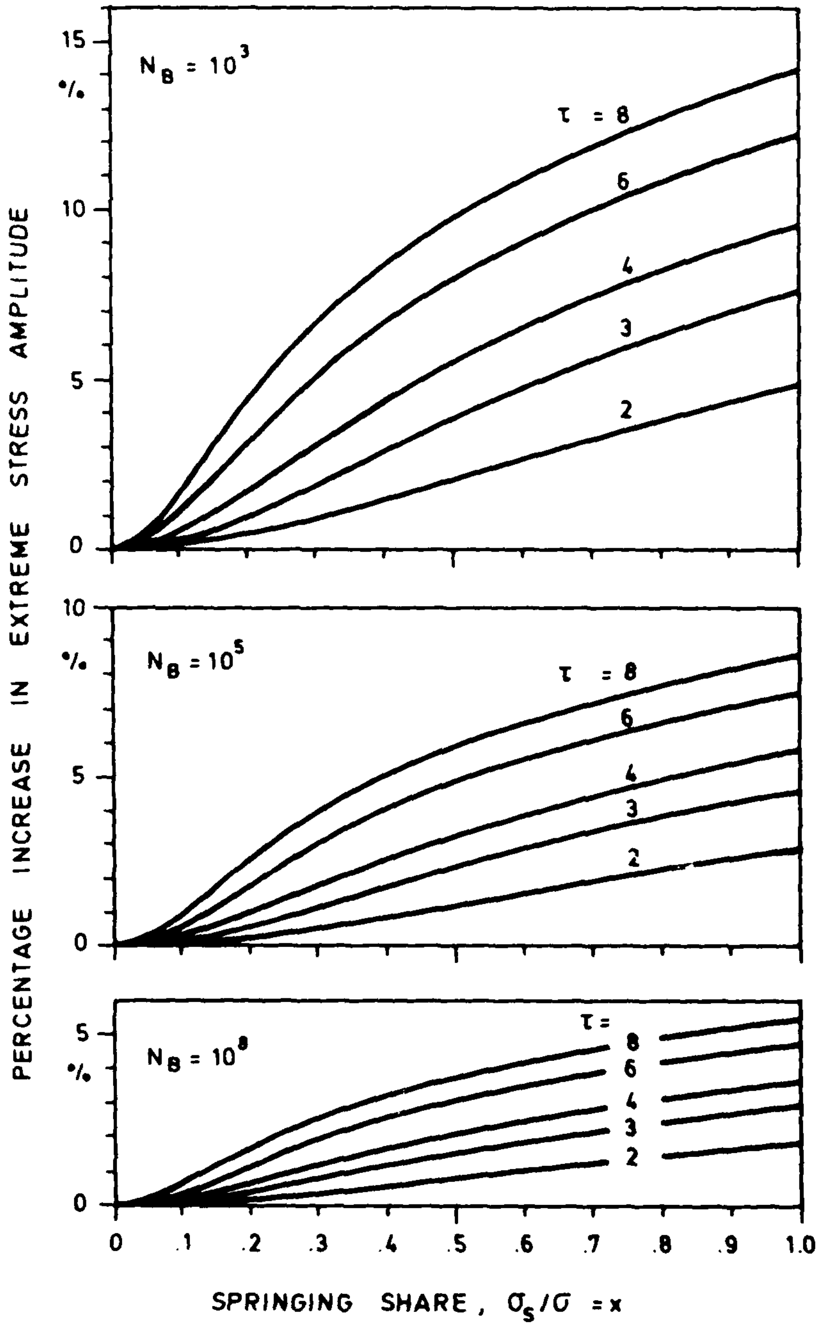

Fig. 4.2.1 Percentage increase in characteristic extreme stress amplitude as a function of the springing share x, by changing bending period (τ) and time interval (N_B).

4.3 Extreme positive maximum

So far the total distribution of both positive and negative maxima has been considered by using the complete Rice distribution described in Section 3.1.

One should also consider the short term extreme value predicted by the positive maxima only. This can be derived from the truncated Rice distribution described in Section 3.2.

In a time interval t there is a number of

$$N_p^+ = aN_p \qquad (4.3.1)$$

positive peaks, as derived in (2.4.3).
A characteristic value for the extreme stress peak is obtained by putting the probability of exceedance in (3.2.5) equal to $1/N_p^+$.
This gives

$$Q^+(Z_c) = \frac{\sqrt{1-\varepsilon^2}}{a} e^{-Z_c^2/2} = \frac{1}{aN_p} \qquad (4.3.2)$$

which gives

$$Z_c = \sqrt{2\ln\sqrt{1-\varepsilon^2}\, N_p} = \sqrt{2\ln N_Z} \qquad (4.3.3)$$

the same as in (4.2.1)
That is: The truncated Rice distribution predicts the same extreme stress under stationary conditions as the complete Rice distribution.

The characteristic extreme value predicted by the generalized gamma distribution may be studied very roughly by considering the equivalent of (4.3.2) introducing an asymptotic expression for Q(Z):

$$\frac{1}{\Gamma(a)}\left(\frac{Z}{\sqrt{2}}\right)^{2a-1/2} e^{-(Z/\sqrt{2})^2}\left\{1 + \frac{a-1}{(Z/\sqrt{2})^2} + \ldots\right\} = \frac{1}{aN_p} \qquad (4.3.4)$$

Putting $1/\Gamma(a) \approx a$ and taking the predominate term of the natural logarithm on each side, one obtains the leading term

$$Z_c = \sqrt{2\ln a^2 N_p} \qquad (4.3.5)$$

which is equivalent to (4.2.1).

For small values of ε we have

$$a^2 = \frac{1}{4}[1 + \sqrt{1-\varepsilon^2}]^2 = \tfrac{1}{2}[(1-\tfrac{1}{2}\varepsilon^2) + \sqrt{1-\varepsilon^2}] \approx \sqrt{1-\varepsilon^2}^2 \qquad (4.3.6)$$

When introduced in (4.3.5) this gives back the previous characteristic extreme (4.2.1), which also appears from Fig. 4.2.1
For $\varepsilon = 1$, on the other side, (4.2.1) breaks together and gives non-sense results while (4.3.5) gives

$$Z_c = \sqrt{2\ \ln(N_p/4)} \qquad (4.3.7)$$

The corresponding expression for $\varepsilon = 1$ derived directly from the one-sided normal distribution is

$$Z_c = \sqrt{2\ \ln N_p/\sqrt{2\pi}} \qquad (4.3.8)$$

which gives about 1.5% higher values, but this equation is still an approximation. Equation (4.3.8) is discussed in /2/.

An expression for the characteristic extreme value which is more complete than (4.3.5), but still an approximation for $\varepsilon \neq 0$ is obtained by taking one term more into consideration in the logarithm of (4.3.4). This gives

$$Z_c = \sqrt{2}\left\{\ln a^2 N_p + \tfrac{1}{2}\sqrt{1-\varepsilon^2}\ \ln(\ln a^2 N_p)\right\}^{\frac{1}{2}} \qquad (4.3.9)$$

but this formula is probably of little practical interest in the present context.

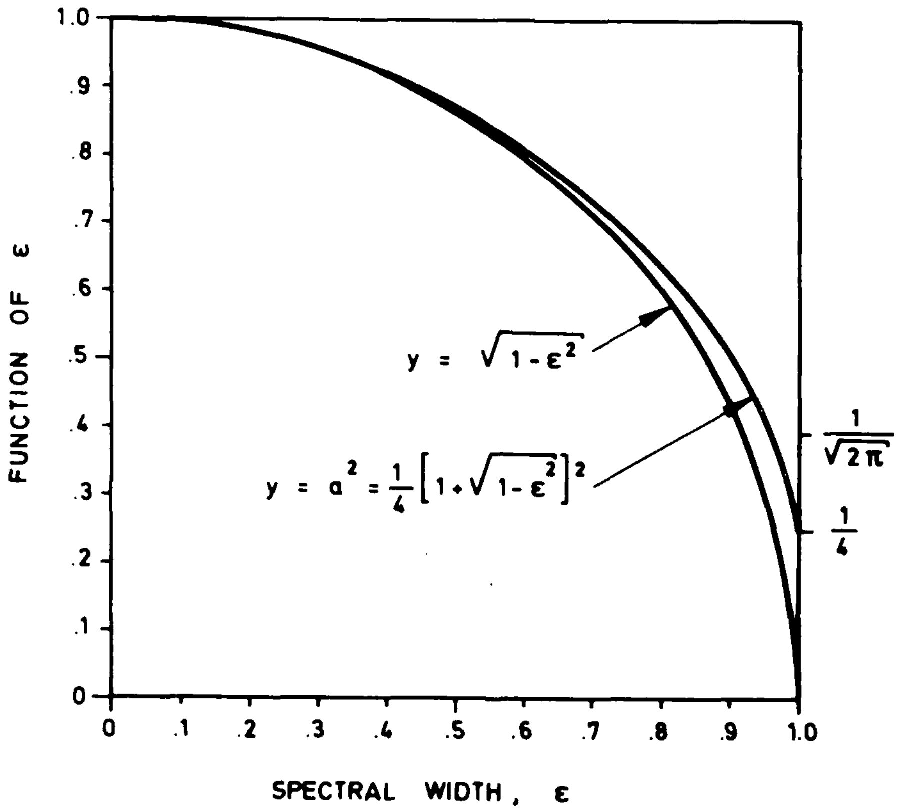

Fig. 4.2.1 Graph of the alternative functions of ε for determination of the effective number of maxima which enters the extreme stress formulae (4.2.1) and (4.3.5), that is

$Zc = \sqrt{2 \ln\sqrt{1-\varepsilon^2}N_p}$, alternatively $= \sqrt{2 \ln a^2 N_p}$.

4.4 Relation to individual maxima

In a given time t the complete signal executes $N_Z = t/T_Z$ zero-up-crossing cycles, and the characteristic extreme stress is

$$S_C = \sqrt{2\sigma^2(\ln t - \ln T_Z)} \qquad (4.4.1)$$

as stated in equation (4.2.1).
In the same time interval the bending stress component executes $N_B = t/T_B$ cycles and has an individual characteristic extreme value

$$S_{CB} = \sqrt{2\sigma_B{}^2(\ln t - \ln T_B)} \qquad (4.4.2)$$

Similarly the springing component executes $N_S = t/T_S$ cycles and has its individual characteristic extreme value

$$S_{CS} = \sqrt{2\sigma_S{}^2(\ln t - \ln T_B)} \qquad (4.4.3)$$

Hence since $\sigma^2 = \sigma_B{}^2 + \sigma_S{}^2$ according to (2.1.2), the individual extremes for bending and springing may be introduced in (4.4.1) and give

$$S_C = \sqrt{S_{CB}{}^2 \frac{\ln t - \ln T_Z}{\ln t - \ln T_B} + S_{CS}{}^2 \frac{\ln t - \ln T_Z}{\ln t - \ln T_S}} \qquad (4.4.4)$$

That is, the total stress extreme is a weighted quadratic sum of the individual extremes. Since we always have

$$T_S < T_Z < T_B \qquad (4.4.5)$$

the springing contribution has slightly greater weight than the bending contribution.
For sufficiently long times t the weighting factors approach unity, and (4.4.4) becomes

$$S_C \approx \sqrt{S_{CB}{}^2 + S_{CS}{}^2} \qquad (4.4.6)$$

This relation gives in most cases a good indication of the importance of springing as far as extreme value is concerned.

5. STATISTICAL EXTREME VALUE DISTRIBUTION

In the present chapter the statistical probability distribution of the short term extreme value will be discussed. The exact probability function is pointed out together with a number of possible approximations. Each distribution function is given an identifier which is referred to in a comparison which is undertaken at the end of the chapter.

5.1 Exact representation (No. 1a)

Consider a time interval t which contains a sequence of N_p stress peaks which are randomly distributed according to a Rice distribution function. The largest stress peak $Z = S/\sigma$ has probability distribution defined exactly by the following formulae:

Cumulative probability P_t:

$$P_t(Z) = [1-Q(Z)]^{N_p} = [\phi(Z/\varepsilon)+\sqrt{1-\varepsilon^2}\ \phi(\frac{\sqrt{1-\varepsilon^2}}{\varepsilon}Z)\,e^{-Z^2/2}]^{N_p} \qquad (5.1.1)$$

Probability density function

$$g_t(Z) = dP_t(Z)/dZ = N_p\ [1-Q(Z)]^{N_p-1}\,g(Z) \qquad (5.1.2)$$

where Q(Z) and g(Z) are given by the equations (3.1.9) and (3.1.3) respectively.
The spectral width ε is given by (2.2.7) and the number of local maxima N_p is given by (2.4.1).

The expectation value is with some approximation

$$E(Z) = \sqrt{2\ln\sqrt{1-\varepsilon^2}\ N_p} + \frac{c}{\sqrt{2\ln\sqrt{1-\varepsilon^2}\ N_p}}\ ,\quad c = 0.5772 \qquad (5.1.3)$$

which is somewhat higher than the characteristic maximum (4.3.3) Fig. 5.1.1 shows how the probability density changes with the springing share $x = \sigma_s/\sigma$. The bending period has been put equal to 5 times the springing period, that is $\tau = 5$, and a time interval of 5000 springing cycles or 1000 bending cycles is considered,

that is of order 2.5 hours. Corresponding spectral width, number of peaks and extreme values appear in Table 5.1.1. It is observed that the difference between characteristic and expectation extreme is insignificant.

Springing share, x	Bending period, τ	Spectral width, ε	Number of peaks, N_p	Characteristic maximum, Zc	Expectation value E(Z)
0	5	0.0	1000	3.717	3.872
0.333	5	0.8993	4380	3.888	4.036
0.666	5	0.7148	4885	4.034	4.177
1.0	5	0.0	5000	4.127	4.267

Table 5.1.1 Change in spectral width, peak number and maximum values with the springing share. The cases correspond to the distributions in Fig. 5.1.1.

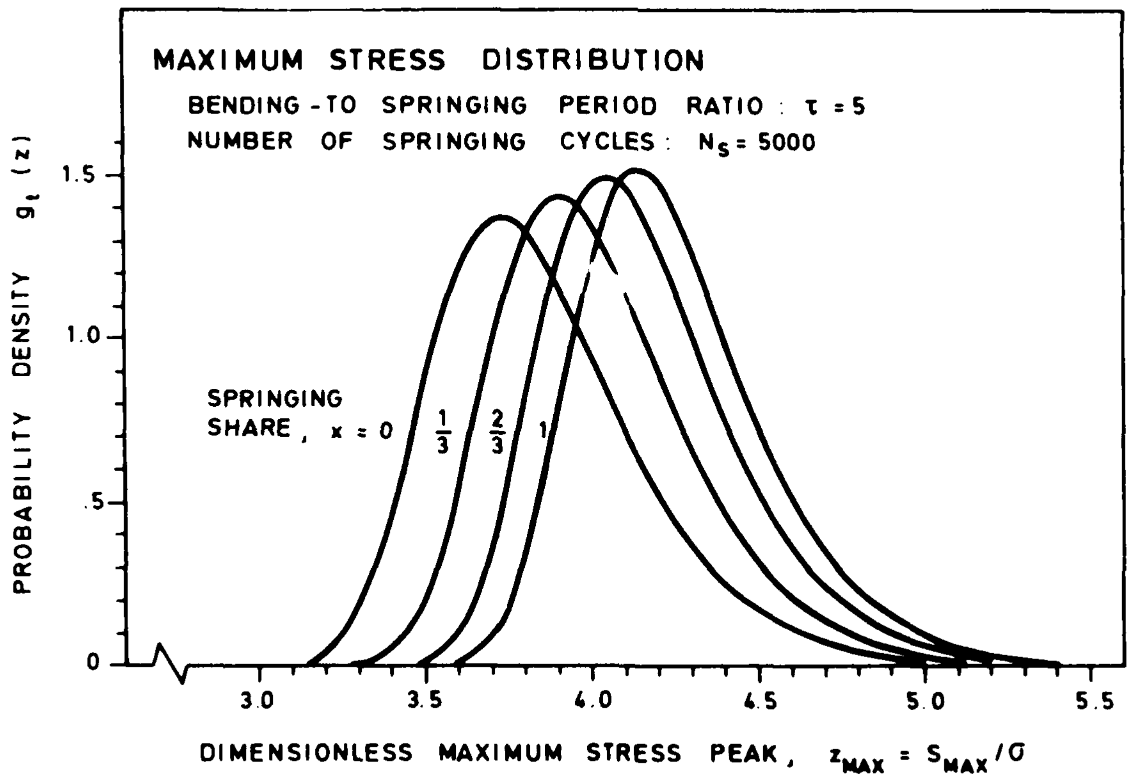

Fig. 5.1.1 Exact probability density function for short time maximum stress.

5.2 Narrow band approximation (No. 2)

The general expressions for the cumulative probability and the probability density were given in (5.1.1) and (5.1.2) in the last article. Introducing Q(Z) from equations (3.1.9) and (3.1.3) respectively, gave the correct distribution functions. One may, however, introduce the low-ε-high-Z approximations (3.1.11) and (3.1.6) instead. This gives the cumulative probability function of the maximum peak

$$P_t(Z) = [1-\sqrt{1-\varepsilon^2}\ e^{-Z^2/2}]^{N_p} \qquad (5.2.1)$$

and the probability density

$$g_t(Z) = N_p[1-\sqrt{1-\varepsilon^2}\ e^{-Z^2/2}]^{N_p-1}\ \sqrt{1-\varepsilon^2}\ Ze^{-Z^2/2} \qquad (5.2.2)$$

where ε and N_p are to be evaluated as before, that is by (2.2.7) and (2.4.1) respectively.

It appears from the discussion in Section 5.7 that this approximation is very close to the exact distribution in the present context, but it breaks of course together for ε approching 1.

5.3 Approximation with double exponential distribution (No. 3a)

The cumulative probability function of the short term extreme stress was with very good approximation given by (5.2.1), that is

$$P_t(Z) = [1-\sqrt{1-\varepsilon^2}\ e^{-Z^2/2}]^{N_p} \qquad (5.3.1)$$

The characteristic value of the extreme value was given in (4.2.1) This may be written as an expression for $\sqrt{1-\varepsilon^2}$, viz. $Z_c{}^2/2$

$$\sqrt{1-\varepsilon^2} = e^{Z_c{}^2/2}/N_p \qquad (5.3.2)$$

This may be introduced in (5.3.1) and gives then

$$P_t(Z) = [1-\frac{1}{N_p}\ e^{-(Z^2-Z_c{}^2)/2}]^{N_p}$$

$$\approx e^{-e^{-(Z^2-Z_c{}^2)/2}} = e^{-N_z e^{-Z^2/2}} \qquad (5.3.3)$$

according to the definition of e.

The probability density function is

$$g_t(Z) = Z\ e^{-(Z^2-Z_c{}^2)/2}\ e^{-e^{-(Z^2-Z_c{}^2)/2}} \qquad (5.3.4)$$

N_p and ε do not appear as individual parameters, but rather through the combination

$$\sqrt{1-\varepsilon^2}\ N_p = N_z = \frac{t}{T_s}\ \sqrt{1 + x^2(\tau^2-1)} \qquad (5.3.5)$$

in the characteristic value

$$Z = \sqrt{2\ \ln N_z} \qquad (5.3.6)$$

The goodness of this approximation is discussed in Section 5.7, and is found to agree very closely with the exact distribution.

5.4 Approximation with square normal distribution (No. 4a,b)

The double exponential approximation for the short term extreme stress, derived in Section 5.3 may be somewhat modified by considering the variable

$$y = (Z^2 - Z_c^2)/2 \qquad \text{or } Z^2 = 2y + Z_c^2 \tag{5.4.1}$$

The probability density function of y is, from (5.3.9),

$$g_t'(y) = e^{-y} \cdot e^{-e^{-y}} = e^{-y} \cdot e^{-1+y-y^2/2+\dots}$$

$$= \frac{1}{e} e^{-y^2/2+\dots} \approx \frac{1}{\sqrt{2\pi}} e^{-y^2/2} \tag{5.4.2}$$

That is, y is approximately normal distributed with expectation 0 and variance 1. Hence the square of the extreme stress Z^2 is approximately normal with expectation Z_c^2 and variance 2.

Hence the probability density of Z is approximately

$$g(Z) = \frac{1}{\sqrt{2\pi}} Z \, e^{-(Z^2-Z_c^2)^2/8} \tag{5.4.3}$$

with the cumulative probability function

$$P_t(Z) = \phi\left(\frac{Z^2-Z_c^2}{\sqrt{2}}\right) \tag{5.4.4}$$

where ϕ is the ordinary normal probability integral.
One may argue that one should apply the expectation value (5.1.3), that is

$$Z_c = 2 \ln N_z + \frac{0.5772}{2\ln N_z} \qquad \text{(0.5772....is Euler's constant)} \tag{5.4.5}$$

rather than the characteristic extreme value (4.2.1) in this connection.
As appears from Section 5.7, the distribution has no skewness, but the variance seems reasonably correct.

It should be mentioned that according to the last section the more exact distribution of the variable y in (5.4.1) is the elementary double exponential distribution with density

$$g(u) = e^{-y} e^{-e^{-y}}$$

which has the following main parameters, exactly determined:

Mean value	$\bar{y}$	=	$\Psi(1) = 0.57721$ (Eulers const.)
2. central moment	$E(y-\bar{y})^2$	=	$\Psi'(1) = 1.64493$
3. central moment	$E(y-\bar{y})^3$	=	$\Psi''(1) = 2.40411$

---etc.-------

The Ψ, Ψ', Ψ''.... functions are the successive derivatives of $\ln\Gamma$.

5.5 Application of distribution for positive maxima (No. 1b)

The probability distribution of the positive maxima was treated in Section 3.2. It was also shown in Section 4.2, equations (4.3.1) to (4.3.3) that this distribution gives the same characteristic extreme value as the complete Rice distribution, provided that the number of peaks is correct. The probability distribution of the extreme stress is then also expected to be closely the same.

The basic equation for the cumulative probability function of the extreme, corresponding to (5.1.1) is

$$P_t^+(Z) = [1 - Q^+(Z)]^{N_p^+}, \qquad \begin{array}{ll} Q^+(Z) & \text{given by (3.2.4)} \\ N_p^+ & \text{given by (3.2.1)} \end{array} \tag{5.5.1}$$

The probability density function is, similar to (5.1.2),

$$g^+(t) = N_p^+ \, [1-Q^+(Z)]^{N_p^+-1} \, g^+(Z), \quad g^+(Z) \text{ given by (3.2.2)} \tag{5.5.2}$$

The distribution is very slightly different from (5.1.2)

For this extreme value probability distribution, namely (5.5.1), we may find the small-ε-large-Z-approximation by application of $Q^+(Z)$ from (3.2.5), just as it was done for the complete distribution in Section 5.2. This gives the cumulative probability distribution

$$P_t^+(Z) = [1- \frac{\sqrt{1-\varepsilon^2}}{a} e^{-Z^2/2}]^{aN_p} \tag{5.5.3}$$

which is not exactly the same as (5.3.1), but indeed very close.

One may, however, proceed further and find the double exponential approximation to (5.5.3), as it was done in Section 5.3. For this purpose one expresses the characteristic extreme from (4.3.2) on the form

$$\frac{\sqrt{1-\varepsilon^2}}{a} = \frac{1}{aN_p} e^{Z_c^2/2} \tag{5.5.4}$$

and introduces this into (5.5.3) This gives

$$P_t^+(Z) = [1-\frac{1}{aN_p} e^{-(Z^2-Z_c^2)/2}]^{aN_p} \approx e^{-e^{-(Z^2-Z_c^2)/2}} \tag{5.5.5}$$

which is identical with (5.3.3).

That is: The extreme value distribution of positive maxima coincides with the complete extreme value distribution on the double exponential distribution level of approximation.

5.6 Approximation with generalized gamma distribution (No. 5)

In Section 3.3 it was shown that the truncated Rice distribution for the positive maxima may be replaced by a generalized gamma distribution with reasonable accuracy. Hence a probability distribution function for the short term extreme value may be established by the basic formulae (5.5.1) and (5.5.2) with application of the required functions from Section 3.3.
This gives the cumulative probability function:

$$P_t^+(Z) = [1 - \Gamma(a;Z^2/2)/\Gamma(a)]^{aN_p} \tag{5.6.1}$$

and the probability density function

$$g_t^+(Z) = aN_p\ [1 - \Gamma(a;Z^2/2)/\Gamma(a)]^{aN_p-1}\ \frac{\sqrt{2}}{\Gamma(a)}\left(\frac{Z}{\sqrt{2}}\right)^{2a-1} e^{-Z^2/2} \tag{5.6.2}$$

with the parameters a and aN_p defined in (2.2.8) and (2.4.1)-(2.4.3) respectively. That is

$$a = \tfrac{1}{2}\left[1 + \frac{1+x^2(\tau^2-1)}{\sqrt{1+x^2(\tau^4-1)}}\right] \tag{5.6.3}$$

and

$$aN_p = N_p^+ = \tfrac{1}{2}[N_p + N_z] = \tfrac{1}{2}\ N_B\left[\sqrt{\frac{1+x^2(\tau^4-1)}{1+x^2(\tau^2-1)}} + \sqrt{1+x^2(\tau^2-1)}\ \right] \tag{5.6.4}$$

The distribution function has very much the same shape, while the most probable extreme is slightly lower.

One may also reconsider the double exponential distribution discussed in Section 5.3, with the characteristic extreme as $\sqrt{2}\ \ln a^2N_p$ derived in (4.3.5). This gives a distribution of correct shape, but with slightly too high mean value.

5.7 Discussion of alternatives

Previous in this article different representations of the probability distribution of the short term extreme stress peak have been derived. These are:

No.1a) The exact probability function based on the complete Rice distribution of the total ensemble of local maxima. Derived in Section 5.1.

No.2 Narrow band approximation of the short term exceedance probability introduced into No.1. Derived in Section 5.2.

No.3a) Double exponential distribution derived from No.2 using the characteristic extreme of the Rice distribution. Derived in Section 5.3.

No.4a) Square normal distribution derived from No.3a) using the characteristic extreme value as a parameter. Derived in Section 5.4 using equation (5.4.5).

No.4b) Square normal distribution derived as in No.4a), but using the expectation value of the extreme as a parameter. Derived in Section 5.4 using equation (5.4.5).

No.1b) Exact probability function based on the truncated Rice distribution for positive maxima. Derived and discussed in Section 5.5.

No.5 Truncated Rice distribution in No.1b) replaced by the generalized gamma distribution approximation. Derived in Section 5.6.

No.3b) Double exponential distribution using the characteristic extreme of the generalized gamma distribution. Derived in Section 5.6.

The different formulae have been compared by considering a particular case selected in the region where approximations are most likely to fail, viz. in the large -short time region.

The case with the following parameters has been chosen:

Springing share	x	$= 1/3$
Bending/springing period ratio	τ	$= 5$
Number of springing cycles	N_s	$= 5000$
Hence:		
Bending share	$\sqrt{1-x^2}$	$= 0.9428$
Spectral width	ε	$= 0.8993$
Peak-to zero crossing period ratio	α	$= 0.4373$
Fraction of positive maxima	a	$= 0.71867$
Number of bending cycles	N_B	$= 1000$
Number of zero crossings ($a^2N_p = 2262.2$)	N_z	$= 1915.5$
Total number of peaks	N_p	$= 4380$
Number of positive peaks	N_p^+	$= 3147.8$
Characteristic extreme from Rice distribution	Z_c	$= 3.8879$
Expectation extreme from Rice distribution	$E(Z_c)$	$= 4.0363$
Characteristic extreme from gamma distribution	Z_c	$= 3.9304$
Alternative value from gamma distribution	$E(Z_c)$	$= 4.0425$

Graphs of the different distributions are shown in Fig. 5.7.1.
A table of the same distributions are given in Table 5.7.1.
A table with a qualitative indication of the fitness of the approximate distributions is given in Table 5.7.2.

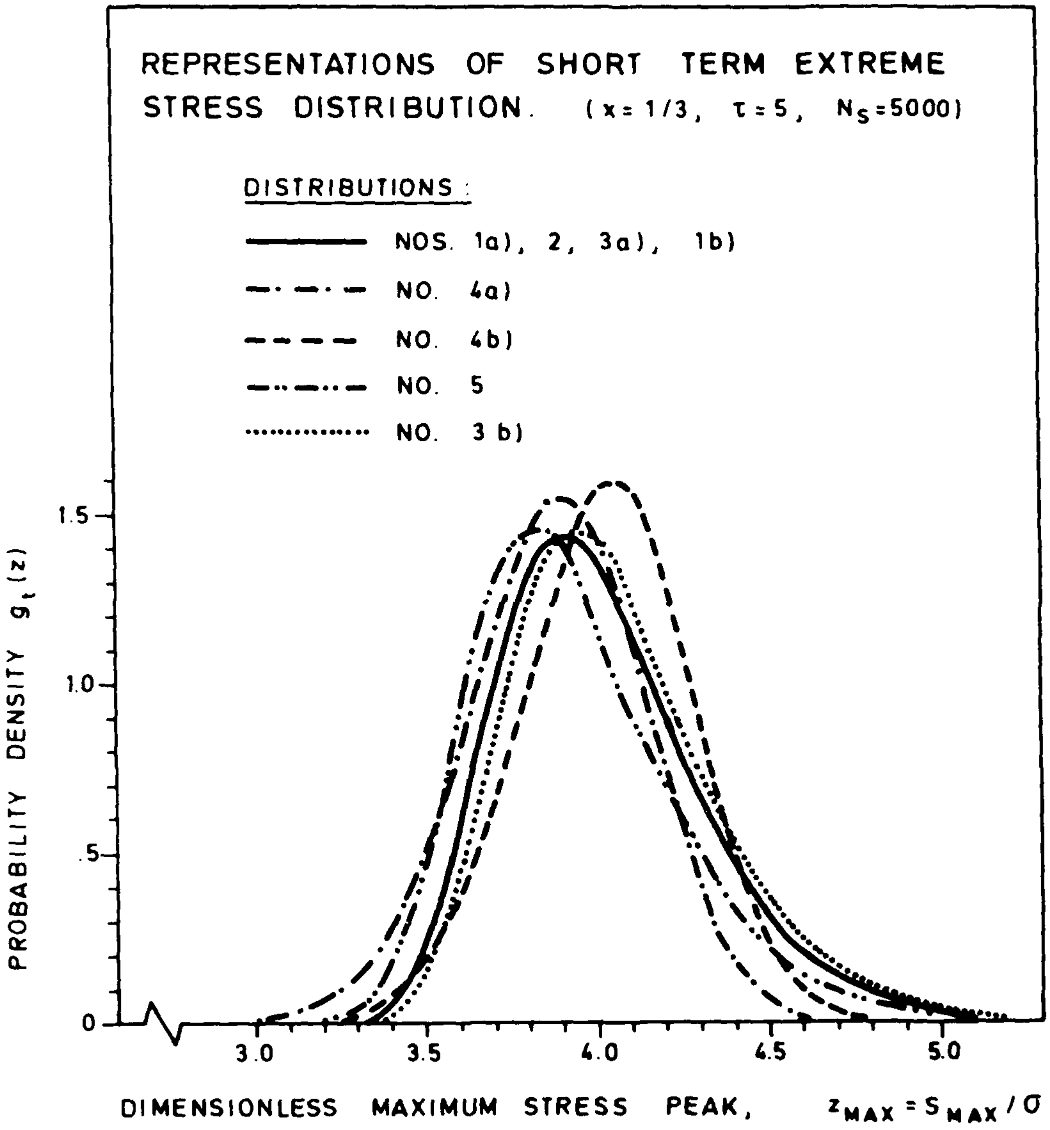

Fig. 5.7.1 Graphs of different representation of the probability density function of the short term extreme in a selected case, $(x = 1/3,\ \tau = 5,\ N_S = 5000)$. See also Fig. 12.1.

No.	1a)	2	3a)	4a)	4b)	1b)	5	3b)
Distr	Exact. Complete Rice	Narrow band approx.	Double exponential Rice extreme	Square normal with characteristic extr.	Square normal with expectation extreme	Exact. Truncated Rice	General gamma	Double exponential Gamma extr.
z Eq	(12.2) (8.9) (3.8)	(12.11)	(12.24) (11.1)	(12.32) (11.1)	(12.32) (12.34)	(12.41) (9.2) (9.4)	(12.51)	(12.24) (11.7)
3·0	0·000000025	0·000000034	0·000000036	0·011162233	0·001554117	0·000000025	0·00000187	9·188226-1[illegible]
3·1	0·000[illegible]6[illegible]45	0·000007311	0·000007493	0·027978173	0·004660877	0·000005984	0·00015761	0·[illegible]517
3·2	0·000343898	0·000386399	0·000391208	0·065409965	0·013114824	0·000342141	0·00389605	0·000058198
3·3	0·006476684	0·006942085	0·006983365	0·141302066	0·034299821	0·006460647	0·03769125	0·0018459[illegible]7
3·4	0·051958155	0·054062626	0·054205966	0·279334205	0·082574674	0·051899582	0·17893840	0·021941200
3·5	0·217170370	0·221858327	0·222091055	0·500298514	0·181169872	0·217069748	0·49747127	0·122665664
3·6	0·554352037	0·560079314	0·560255669	0·803507663	0·358538181	0·554267172	0·92844585	0·388779537
3·7	0·977948458	0·981659760	0·981668840	1·144987516	0·633270666	0·977929988	1·29092104	0·801492074
3·8	1·311165970	1·311353771	1·311228237	1·431936489	0·987438867	1·311207568	1·44665308	1·201552501
3·9	1·435856679	1·433316799	1·433153522	1·554138478	1·344090850	1·435921557	1·38515818	1·424179125
4·0	1·355552065	1·351955553	1·351820929	1·447097378	1·578860383	1·355611489	1·18296846	1·421201284
4·1	1·148170113	1·144785552	1·144697534	1·142393187	1·581696828	1·148213560	0·92964499	1·259970276
4·2	0·898337671	0·895715636	0·895665949	0·755404671	1·335068603	0·898365880	0·68760466	1·004949638
4·3	0·663051603	0·661245010	0·661219660	0·413233135	0·937756194	0·663068841	0·46630103	0·755171790
4·4	0·468636480	0·467484687	0·467472668	0·184644378	0·541198257	0·468646752	0·33255248	0·540239095
4·5	0·320543554	0·319847972	0·319842583	0·066518973	0·253306272	0·320549652	0·22157360	0·372517793
4·6	0·213740846	0·213337036	0·213334722	0·019064943	0·094879117	0·213744490	0·14463459	0·249733810
4·7	0·139646268	0·139418752	0·139417793	0·004288319	0·028055084	0·139648465	0·09281727	0·163740893
4·8	0·089702079	0·089576888	0·089576502	0·000746535	0·006458311	0·089703414	0·05869921	0·105424612
4·9	0·056781745	0·056714191	0·056714040	0·000099162	0·001141070	0·056782559	0·03664163	0·066836395
5·0	0·035474326	0·035438482	0·035438425	0·000009905	0·000152503	0·035474822	0·0226[illegible]09	0·041798065
5·1	0·021895619	0·021876886	0·021876864	0·000000733	0·0000151[illegible]	0·021895921	0·01378266	0·025816023
5·2	0·013360536	0·013350880	0·013350872	0·000000039	0·000001110	0·013360718	0·00831359	0·015759782
5·3	0·008063042	0·008058131	0·008058128	0·000000001	0·000000058	0·008063151	0·00496238	0·009513873
5·4	0·004813945	0·004811479	0·004811478	4·220030-11	0·000000002	0·004814010	0·00293133	0·005681346
5·5	0·002843849	0·002842626	0·002842625		5·816873-11	0·002843887	0·00171381	0·003356772
5·6	0·001662508	0·001661909	0·001661909		1·055472-12	0·001662530	0·00099177	0·001962577
5·7	0·000961843	0·000961553	0·000961553			0·000961856	0·00056811	0·001135541
5·8	0·000550742	0·000550603	0·000550603			0·000550749	0·00032213	0·000650241
5·9	0·000312111	0·000312046	0·000312046			0·000312116	0·00018081	0·000368517
6·0	0·000175064	0·000175034	0·000175034			0·000175067	0·0001[illegible]46	0·000206711

Table 5.7.1 Different representations of the short term extreme value distribution. $x = 1/3$. $\tau = 5$. $N_S = 5000$.

DISTRIBUTION		Probable largest value	Standard deviation	Skewness
1a)	Exact. Complete Rice distribution	Basis for comparison	Basis for comparison	Basis for comparison
2	Narrow band approximation	++	++	++
3a)	Double exponential Rice extreme	++	++	++
4a)	Square normal, characterist. extr.	+	+	Skewness is too small
4b)	Square normal expectation extreme	3.5% high	+	Skewness is too small
1b)	Exact. Truncated Rice distribution	++	++	++
5	General gamma distribution	2% low	++	+
3b)	Double exponential gamma extreme	1% high	++	++

++ very close agreement

\+ reasonably close agreement

Table 5.7.2 Tentative evaluation of the three first moments of the extreme value distribution representations. The evaluation has been performed by inspection of Fig. 5.7.1. The case is realistic but unfavourable. Discrepancies are generally lower.

5.8 Dispersion of the extreme value distribution

Fractiles of the short term extreme value distribution can conveniently be evaluated from the double exponential distribution representation, equation (5.3.3)

$$P = e^{-N_Z e^{-Z^2/2}} \tag{5.8.1}$$

Solving for Z gives immediately the P-fractile Z_P

$$Z_P = \sqrt{-2 \ln(\frac{1}{N_Z} \ln \frac{1}{P})} = \sqrt{2} \sqrt{\ln N_Z - \ln(\ln \frac{1}{P})} \tag{5.8.2}$$

That is: There is 100·P% chance that the extreme value shall be less or equal to Z_P.
The characteristic value Z_C correspond to the 36.8% fractile (1/e).
As a measure for dispersion we may choose the 68% confidence interval (corresponding to the ± one standard deviation in the normal distribution). The interval is determined by:

- Lower limit, 16% fractile

$$Z_{0.16} = \sqrt{2} \sqrt{\ln N_Z - 0.606} \tag{5.8.3}$$

- Upper limit, 84% fractile

$$Z_{0.84} = \sqrt{2} \sqrt{\ln N_Z + 1.75} \tag{5.8.4}$$

Taking half of the 68% confidence interval as an estimate for the standard deviation of the extreme value δz, we find

$$\delta Z = \frac{1}{\sqrt{2}} \left\{ \sqrt{\ln N_Z + 1.75} - \sqrt{\ln N_Z - 0.606} \right\} \tag{5.8.5}$$

The relative deviation, $\delta Z/Z_C$, is shown in percent in Fig. 5.8.1 and is seen to have characteristic values of order 5-10%.
This is in the same order of magnitude as the increase experienced when the stress goes from pure bending to pure springing (Fig. 4.2.1)

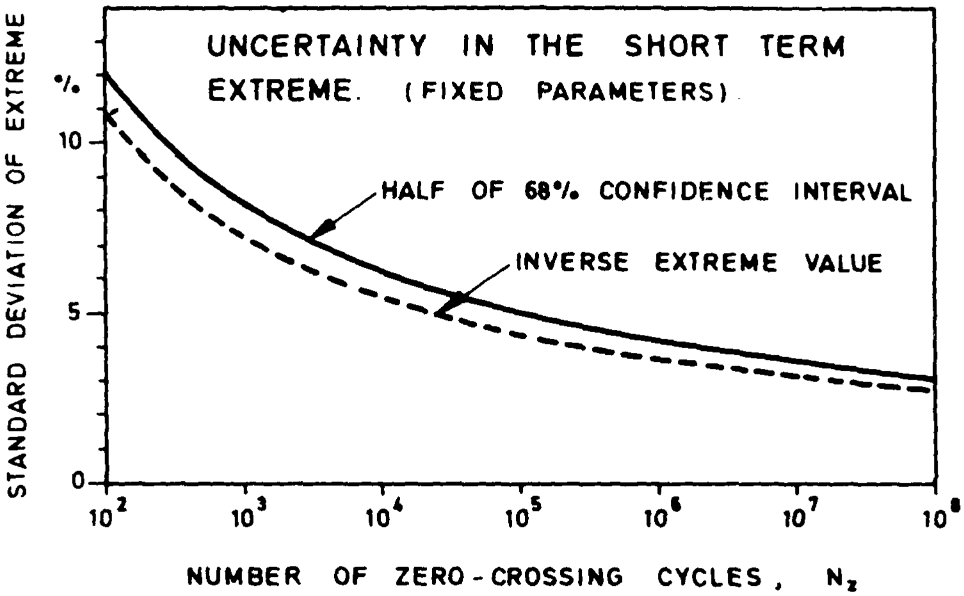

Fig. 5.8.1 Standard deviation of the extreme value represented by one half of the 68% confidence interval (heavy line) and by $1/Z_c$ (dotted line). The deviation cover only the natural dispersion present when all parameters are known and fixed.

A more handy but less stringent expression for the dispersion of the extreme value can be derived from Section 12.4. In this section it was shown that the variable

$$y = (Z^2 - Z_c^2)/2 \tag{5.8.6}$$

is approximately normal distributed with expectation 0 and standard deviation 1. Since Z is rather close to Z_c, (13.6) gives

$$y = \tfrac{1}{2}(Z - Z_c)(Z + Z_c) \approx \tfrac{1}{2}(Z - Z_c)2Z_c = (Z - Z_c)/(1/Z_c) \tag{5.8.7}$$

which shows that Z is roughly normal with standard deviation

$$\delta Z = 1/Z_c \tag{5.8.8}$$

The relative standard deviation, $1/Z_c{}^2$, is plotted in Fig. 5.8.1. It is found to correspond closely to 1/2 of the 60% confidence interval.

6. EXTREME VALUE DISTRIBUTION BY UNKNOWN SPECTRAL WIDTH

So far we have only considered stationary cases where the springing share x and the bending period ratio τ are assumed to be known. When x and τ are known, one also knows the spectral width ε and the related parameters α and a which in turn determine the number of zero crossings N_Z (in terms of springing cycles), and N_Z is the only term in the basic extreme value expression which is dependent of the springing-to-bending relationships. See equations (4.2.1), (4.3.3), (5.1.3), (4.3.5), (4.3.9) among others.

Further about the number of zero crossing cycles, we know that it will always lie somewhere between the number of bending cycles N_B and the number of springing cycles N_S, thus

$$N_B \leqslant N_Z \leqslant N_S \quad \text{or} \quad \frac{1}{\tau} \leqslant \frac{N_Z}{N_S} \leqslant 1 \tag{6.1.1}$$

This was discussed in Section 2.4.

Thus, in a stationary condition, if one knows the total stress RMS value σ, the springing and bending periods, N_S and τ, but is completely ignorant about the mixing ratio of springing and bending, the number of zero crossing cycles N_Z can only be determined by a probability distribution. The probability distribution which conveys minimum information and which introduces largest uncertainty in the predicted extreme value is the uniform distribution between N_B and N_S. That is: N_Z has the probability density function

$$h(N) = \frac{1}{N_S - N_B} \qquad N_B \leqslant N \leqslant N_S \tag{6.1.2}$$

which has the mean value

$$\bar{N} = \tfrac{1}{2}(N_B + N_S) \tag{6.1.3}$$

and the standard deviation

$$\delta_N = \left\{\frac{1}{3}\,\frac{N_S{}^3 - N_B{}^3}{N_S - N_B} - \frac{1}{4}(N_S + N_B)^2\right\}^{\frac{1}{2}} \tag{6.1.4}$$

Expressed by the bending period ratio τ, the standard deviation relative to the mean value is

$$\frac{\sigma N}{\bar{N}} = \left\{ \frac{4}{3} \frac{\tau^3 - 1}{(\tau-1)(\tau+1)^2} - 1 \right\}^{\frac{1}{2}} \tag{6.1.5}$$

A plot is given in Fig. 6.1.1.

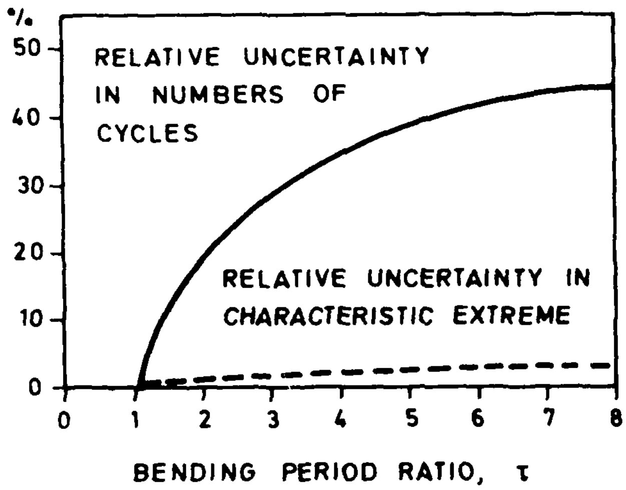

Fig. 6.1.1 Relative uncertainty in number of zero-crossing cycles in the case of a uniform probability distribution between N_B and N_S. The dotted line is the corresponding uncertainty in the characteristic extreme value.

In the case studied in Section 12.7 a time interval was considered which covered N_S = 5000 springing cycles and N_B = 1000 bending cycles, that is = 5. If one is ignorant about the number of zero crossings in this case, the uniform distribution (6.1.2) prescribes a mean number of (3000 $\pm$ 1154) cycles. The uncertainty is 38.5%.

With this probability distribution of N_Z, the characteristic extreme value

$$Z_c = \sqrt{2 \ln N_Z} \tag{6.1.6}$$

also becomes a random variable with a probability density function

$$k(Z) = \frac{1}{N_S - N_B} \frac{1}{2} Z \; e^{Z^2/2}, \quad \sqrt{2 \ln N_B} < Z < \sqrt{2 \ln N_S} \tag{6.1.7}$$

The relative uncertainty in the characteristic extreme caused by the uncertainty in number of cycles is approximately

$$\frac{\delta Z}{\bar{Z}} \approx \frac{1}{2} \frac{1}{\ln N} \frac{\delta N}{\bar{N}} \approx \frac{1}{\bar{Z}^2} \frac{\delta N}{\bar{N}} \tag{6.1.8}$$

the relative uncertainty in N being given by (6.1.5). Since N in the short term case is of order N = 1000 we have (2lnN) being roughly 15. The relative uncertainty in the characteristic extreme is thus of order 1/15 of the relative uncertainty in the number of periods, which is below 5% in most practical cases. In the numerical example it is about 2.5%. This means in the practice that the extreme value distribution under uniformly distributed number of cycles is expected to deviate insignificantly from the extreme value distribution obtained with the number of cycles fixed at the mean value (6.1.3).

To derive the actual probability distribution of the short term extreme value by unknown number of cycles, the double exponential approximation (5.3.3) is most convenient. The cumulative probability distribution by given N_Z is

$$P_t(Z;\, N_Z = N) = e^{-N e^{-Z^2/2}} \tag{6.1.9}$$

Weighted by the probability distribution (6.1.2) for the number of cycles N, one obtains

$$\begin{aligned} P_t(Z) &= \frac{1}{N_S - N_B} \int_{N_B}^{N_S} e^{-N e^{-Z^2/2}} \, dN \\ &= \frac{1}{N_S - N_B} \; e^{Z^2/2} \left[e^{-N_B e^{-Z^2/2}} - e^{-N_S e^{-Z^2/2}} \right] \end{aligned} \tag{6.1.10}$$

which is the cumulative probability distribution of the short term extreme when number of zero crossings is unknown. The corresponding probability density is

$$g_t(Z) = \frac{Z}{N_S-N_B}\left\{(N_B+e^{Z^2/2})e^{-N_Be^{-Z^2/2}} - (N_S+e^{Z^2/2})e^{-N_Se^{-Z^2/2}}\right\} \quad (6.1.11)$$

This distribution is graphed in Fig. 6.1.2 for the case studied in Fig. 5.7.1.

Fig. 6.1.2 also shows the probability distribution of the extreme value when the number of zero crossings is fixed at the mean value of N_Z = 3000. The peak values of the two distributions indicate that the standard deviation of the extreme increase by a factor 1.47/1.36 = 1.081, that is 8.1%, by the loss of information about the number of zero crossing cycles.

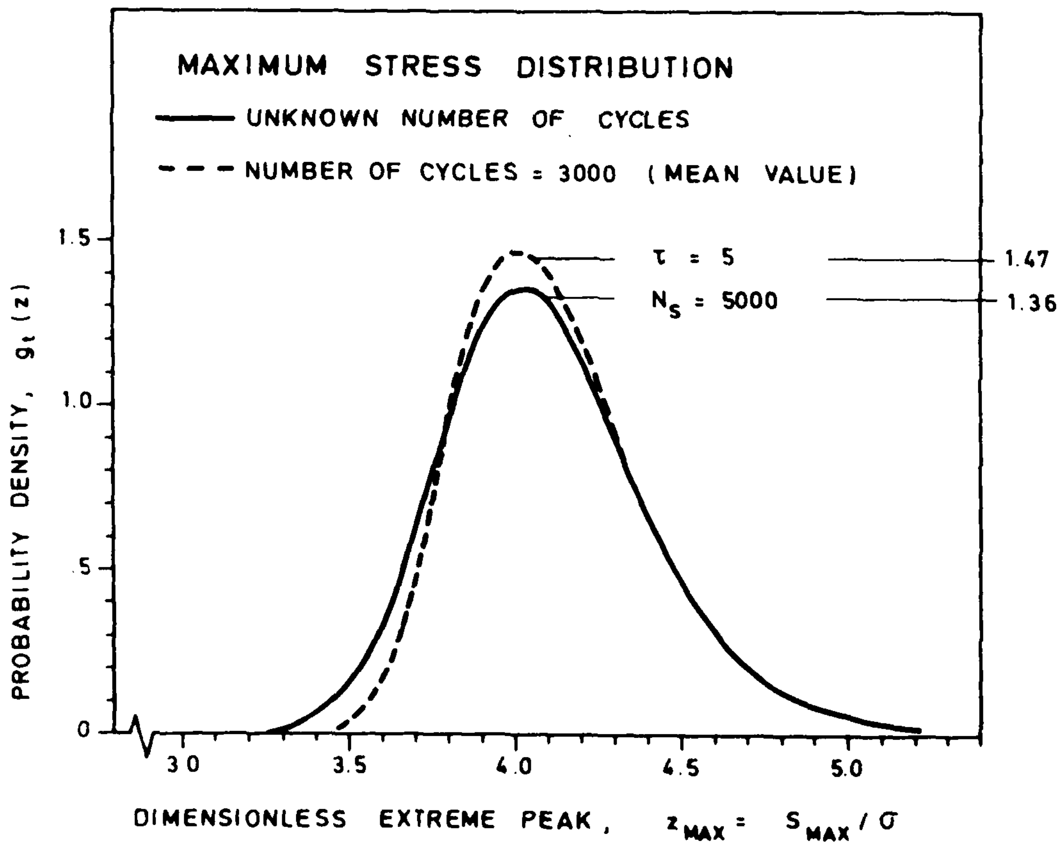

Fig. 6.1.2 Probability function of the extreme by unknown, uniformly distributed number of cycles.

7. UNCERTAINTIES IN SHORT TERM EXTREME VALUE PREDICTIONS

It appears from (4.2.1) and (5.4.1) that the extreme response under stationary conditions can be written

$$S = \sigma Z = \sqrt{2}\sigma\sqrt{\ln N_Z + y} \qquad (7.1.1)$$

y is a normalized random variable which is responsible for the natural dispersion of the extreme value, approximately normal distributed with expectation value $\bar{y} = 0$ and standard deviation $\delta_y = 1$. More exactly y is double exponential distributed with $\bar{y} = 0.5772$ and $\sigma_y = 1.2825$.
N_Z is the number of zero crossings, in the most unfavourable case uniformly distributed between N_B and N_S. Expectation value $\bar{N}$ is the arithmetic mean, and standard deviation δ_N which may amount to the order of N_B (6.1.4)

σ is the RMS of the complete response subjected to a normal error distribution with expectation $\bar{\sigma}$ and standard deviation δ_σ. Depending on the situation, σ may be monitored in real time with a relative uncertainty of order 5-10%, or it may appear from wave load response calculations and has then a still higher uncertainty.

Uncertainties in y, N_Z and σ makes the actual extreme peak S uncertain with expectation

$$\bar{S} = \sqrt{2}\bar{\sigma}\sqrt{\ln \bar{N}} = \bar{\sigma}\bar{Z} \qquad (7.1.2)$$

and standard deviation s given by

$$\frac{\delta_S}{\bar{S}} = \sqrt{\left(\frac{\delta_\sigma}{\bar{\sigma}}\right)^2 + \left(\frac{1}{\bar{Z}^2}\frac{\delta_N}{\bar{N}}\right)^2 + \left(\frac{1}{\bar{Z}^2}\right)^2} \qquad \bar{Z} = \sqrt{2\ln\bar{N}} \qquad (7.1.3)$$

The second term under the square root stems from (6.1.8) and the last term from (5.8.8).

Considering the case studied in section 5.7 and Fig. 6.1.2, and assuming 10% uncertainty in the RMS value σ, we have

$$\begin{aligned} \bar{Z} &= 4.0 \\ \bar{N} &= 3000 \\ \delta_N &= 1154 \end{aligned} \tag{7.1.4}$$

which give

$$\frac{\delta_S}{\bar{S}} = \sqrt{0.10^2 + 0.0240^2 + 0.0625^2} = \sqrt{0.1^2 + 0.067^2} = \underline{0.12} \tag{7.1.5}$$

It is hence seen that the uncertainty caused by unknown number of cycles is nearly exhausted by the natural dispersion y. And both these error sources are nearly exhausted by the uncertainty in the RMS.

It is also observed that when uncertainty in RMS is disregarded, the relative uncertainty in the extreme stress becomes

$$\frac{\delta_S}{\bar{S}} = \frac{1}{\bar{Z}^2} \sqrt{1 + (\frac{\delta_N}{\bar{N}})^2} = \frac{1}{16} \sqrt{1 + 0.15} \approx \frac{1}{16} (1 + 0.075) = 0.067 \tag{7.1.6}$$

which confirms the conclusion drawn from Fig. 6.1.2: By loss of information about the number of cycles, the uncertainty in extreme value increases with 7.5% (≈ 8.1%), viz. by increasing from 0.0625 to 0.067.

8. FATIGUE CRACK AND PROPAGATION

8.1 Elementary considerations

It is convenient to consider fatigue in terms of crack propagation velocity. Consider a crack of present linear extension a. Under the influence of a pure stationary bending stress, the crack proceeds with a mean velocity v_B

$$v_B = \frac{da}{dt} = \frac{da}{dN} \cdot \frac{dN}{dt} = \frac{da}{dN} \cdot \frac{1}{T_B} = \frac{da}{dN} \frac{1}{\tau T_S} \qquad (8.1.1)$$

Under a pure stationary springing stress, the crack proceeds with a mean velocity v_S

$$v_S = \frac{da}{dN} \cdot \frac{1}{T_S} \qquad (8.1.2)$$

Now da/dN which occurs in both (8.1.1) and (8.1.2) is the mean crack increase per cycle, and provided that the stress RMS is the same in both cases, then da/dN are also the same in both cases.

Thus we obtain under very general conditions, that when the stress goes from pure bending to pure springing under fixed RMS, cracks accelerates with a factor

$$\frac{v_S}{v_B} = \frac{T_B}{T_S} = \tau \qquad (8.1.3)$$

and correspondingly, the fatigue life time is reduced by a factor $1/\tau$.
As τ is of order 3-4-5, one may conclude that fatigue may be rather sensitive to the presence of springing.

8.2 Fatigue under combined bending and springing

Proceeding with development of cracks as a model for fatigue and deteriorating processes it can be shown that distinctly separated spectral stress peaks do not interact. This is only an assumption, but it may be supported by results obtained with the rainflow cycle counting method which is presently regarded as the most reliable cycle counting method for crack propagation as well as Miner calculations.
The propagation speed v of a crack is thus the sum of the bending term v_B and the springing term v_S. The linear extension a of a one-dimensional crack has then the velocity

$$v = \frac{da}{dt} = v_B + v_S = \frac{da}{dN}\frac{1}{T_B} + \frac{da}{dN}\frac{1}{T_S} \qquad (8.2.1)$$

According to Paris et.al. /8/ the increment da/dN per cycle is related to the stress intensity ΔK through

$$\frac{da}{dN} = C(\Delta K)^m \qquad \text{(C and m are constants)} \qquad (8.2.2)$$

The stress intensity has a linear relationship to the nominal, local stress amplitude S through

$$\Delta K = \sqrt{\pi a}\; g(a) S \qquad (8.2.3)$$

where g(a) is a geometry factor. When the sequence of amplitudes S is not constant in magnitude, but fluctuating according to a Rayleigh distribution with parameter $\sqrt{2}\sigma$, the average contribution from each cycle is

$$\frac{da}{dN} = C[\sqrt{2\pi a}\; g(a)]^m \; \Gamma(1+m/2)\sigma^m = C'\sigma^m \qquad (8.2.4)$$

C' is a new constant which changes only slowly with the crack depth, but which is instantaneously the same for springing and bending.

Introduced in (8.2.1) we then find the crack velocity

$$v = C[\sqrt{2\pi a}\ g(a)]^m\ \Gamma(1+m/2)\left[\frac{\sigma_B{}^m}{T_B} + \frac{\sigma_S{}^m}{T_S}\right] \qquad (8.2.5)$$

where σ_B and σ_S are the RMS of bending and springing only. Introducing the total RMS, σ, gives

$$v = C[\sqrt{2\pi a}\ g(a)]^m\ \Gamma(1+m/2)\ \frac{\sigma^m}{T_B}\left[(\sqrt{1-x^2})^m + \tau x^m\right] \qquad (8.2.6)$$

Here the last bracket is a factor which tells how more faster the cracking process goes on when the stress changes gradually from pure bending to pure springing through x. The fatigue life is reduced by the same factor. By definition, this factor is closely related to a spectral correction factor, denoted λ, appearing in the literature /9/. For the present case we have thus in particular

$$\lambda' = (\sqrt{1-x^2})^m + \tau x^m \qquad (8.2.7)$$

which is graphed in Fig. 8.2.1 for different values of m and τ. The normal value of m is 3-4, but values up to 8 appear in different codes /10/. (It should be noted that m in the crack propagation approach is equal to the slope parameter of the Wöhler curves usually entering into the Miner fatigue calculations).

It is observed from Fig. 8.2.1 that the fatigue decreses slightly for a small component of springing when the total RMS is given. This is in accordance with common evidence, since fatigue is known to decrease by less regular cycle forms. For larger springing share, the fatigue rate increases rapidly up to the factor τ predicted in Section 8.1 due to the increased number of cycles. In case the squared springing share x^2 in the long run is Beta distributed (see section 9.2) with parameters (r,s), and τ is considered constant, the long term average of the spectral correction factor due to springing is

$$\overline{\lambda'} = \frac{1}{B(r,s)}\left[B(r,s+m/2) + \tau B(r+m/2,s)\right] \qquad (8.2.8)$$

There is one inconsistency in (8.2.7): The spectral correction factor λ should approach 1 for $\tau=1$, that is when the two spectral components coincide to one. This is only the case for m=2.

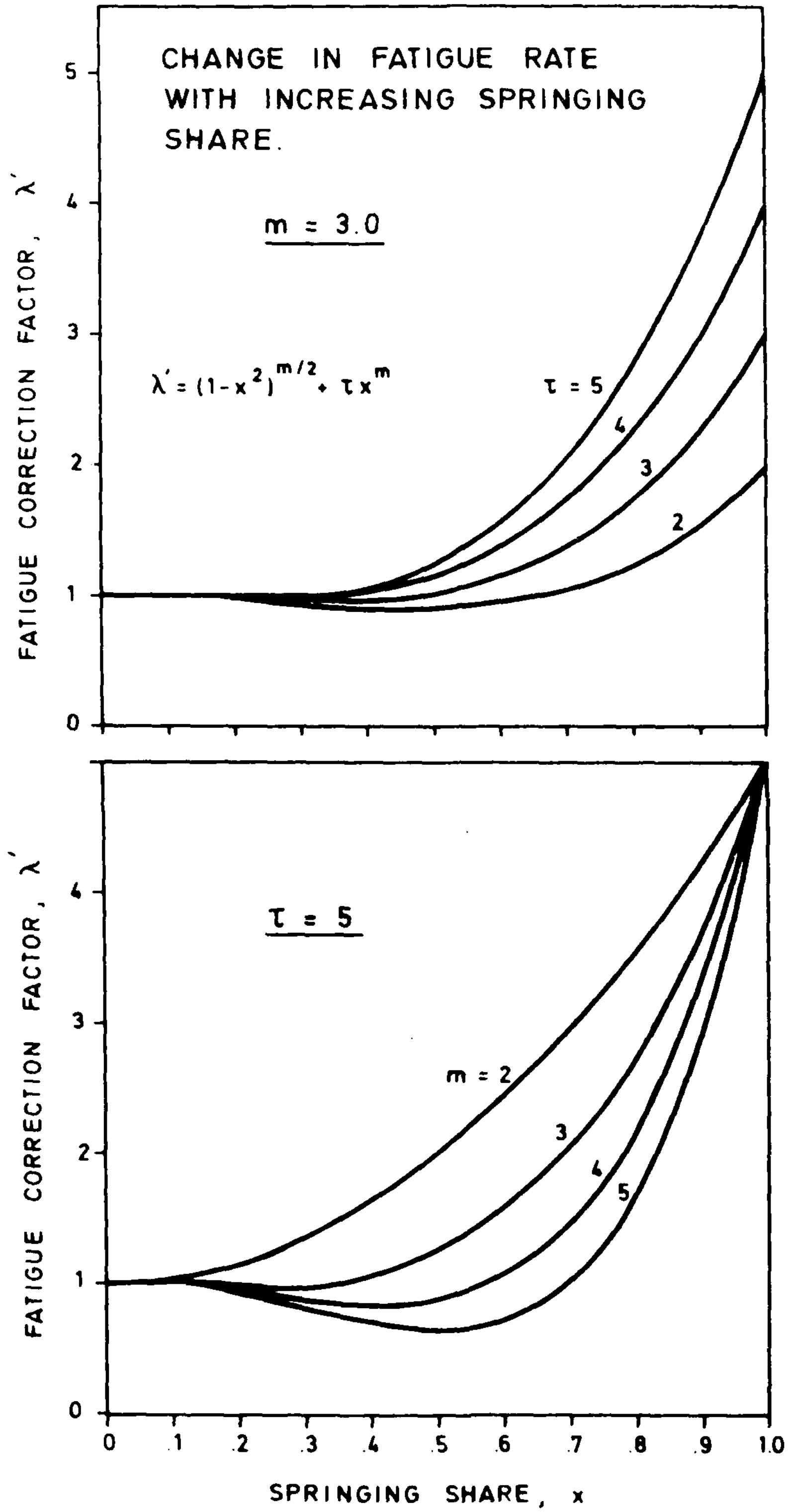

Fig. 8.2.1. Spectral correction factor for fatigue by fixed σ and T_B.

8.3 Comparison with regression formula

An empirical formula for the spectral correction factor λ is suggested in /9/ and has the form

$$\lambda(m,\varepsilon) = a(m) + [1-a(m)](1-\varepsilon)^{b(m)} \tag{8.3.1}$$

By regression analysis of counting tests on simulated random records, the functions were determined to

$$\begin{aligned} a(m) &= 0.926 - 0.033\ m \\ b(m) &= -2.323 + 1.587\ m \end{aligned} \tag{8.3.2}$$

Due to definition of periods, (8.3.1) is not quite the same as (8.3.1), but a relationship may be established by

$$\lambda' = \frac{T_B}{T_Z}\lambda = \sqrt{1+x^2(\tau^2-1)}\ \left\{a(m)+[1-a(m)][1-\varepsilon(x,\tau)]^{b(m)}\right\} \tag{8.3.3}$$

$$\varepsilon(x,\tau) = x(\tau-1)\sqrt{\frac{1-x^2}{1+x^2(\tau^4-1)}} \tag{8.3.4}$$

ε is quoted from (2.2.7).

Tha fatigue correction factor derived from this equation is graphed in Fig. 8.3.1, and should be comparable with Fig. 8.2.1. The fatigue rate is, however, seen to increase much more steadily with the springing share than predicted previously.

The stress spectra on which (8.3.1) and (8.3.2) are based are of different nature than the two-peak spectrum underlying (8.2.7), and it is not immediately clear which procedure that gives the most correct results.

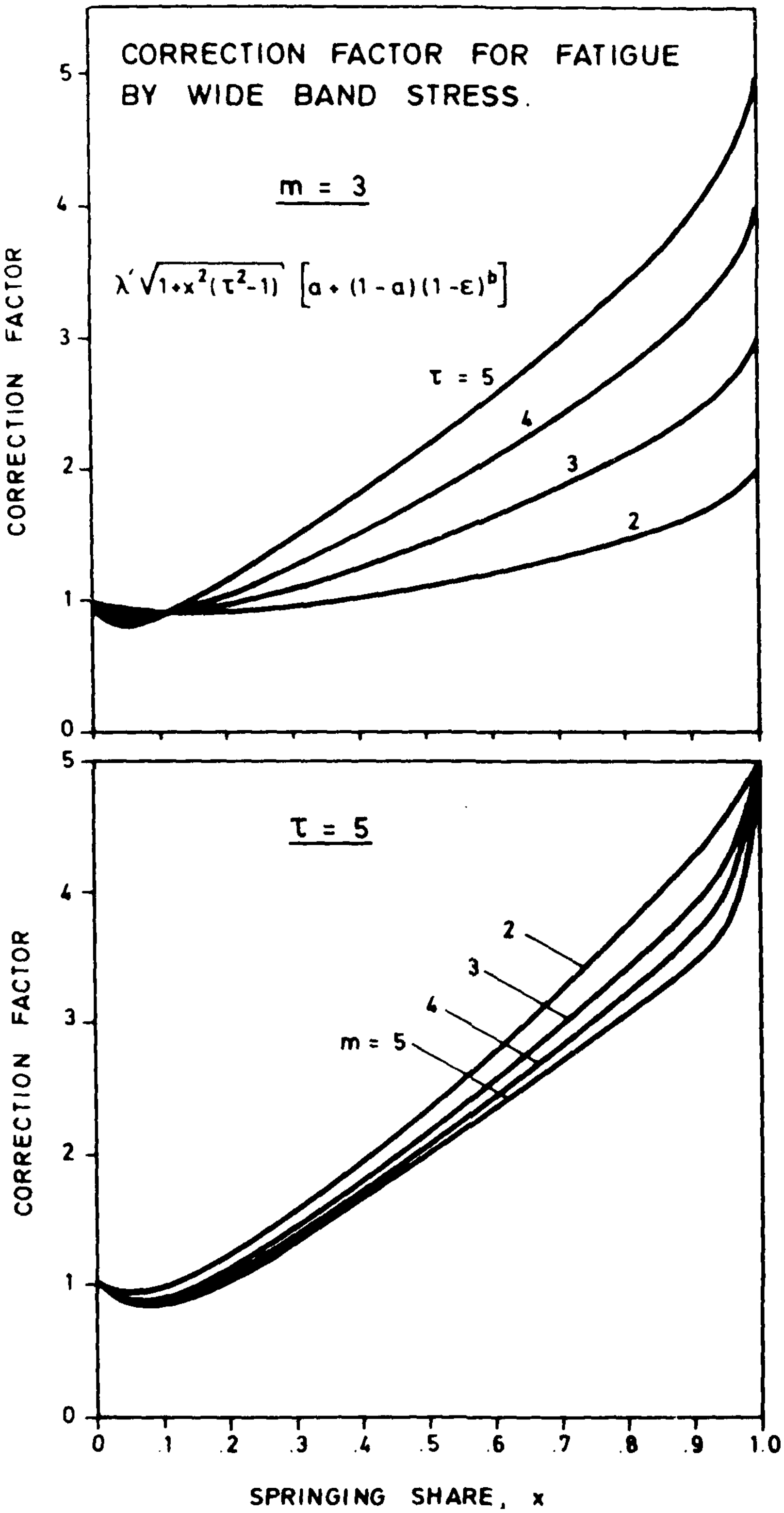

Fig. 8.3.1. Spectral correction factor derived from Wirsching's formula (8.3.3).

9 LONG TERM DISTRIBUTION OF RMS

9.1 Distributions in a special case

In one particular case the long term probability distributions of the total RMS and the springing share can be exactly derived from the long term bending and springing RMS-distributions. This occurs when the bending RMS σ_B follows a general gamma distribution (3.3.1) on the particular form

$$f(\sigma_B) = f(m,\ 2,\ B;\ \sigma_B) = \frac{2}{\Gamma(m)B}\left(\frac{\sigma_B}{B}\right)^{2m-1} e^{-(\sigma_B/B)^2} \tag{9.1.1}$$

and the springing RMS σ_S follows the almost similar distribution

$$f(\sigma_S) = f(n,\ 2,\ B;\ \sigma_S) \tag{9.1.2}$$

In this case the total RMS σ defined through (2.1.2)

$$\sigma^2 = \sigma_B{}^2 + \sigma_S{}^2 \tag{9.1.3}$$

has the related distribution function

$$f(\sigma) = f(m+n,\ 2,\ B;\ \sigma) = \frac{2}{\Gamma(m+n)}\left(\frac{\sigma 1}{B}\right)^{2(m+n)-1} e^{-(\sigma/B)^2} \tag{9.1.4}$$

That is, in this particular case the shape parameters m and n for the bending and springing are additive such that the corresponding parameter for the total RMS is $(m + n)$.

Some selected members of this class of distributions are shown in Fig. 9.1.1, normalized to scale parameter $B = 1$. This plot may in some situations be used qualitatively to judge the importance of vibration components.
For example, if a Weibull plot of the bending and springing RMS are relatively positioned roughly as the curves for $m = 1.0$ and $m = 0.25$ respectively, then the total RMS is positioned roughly as the $m = 1.25$ curve. That is the presence of springing increases the extreme stress level with order 3-4%.

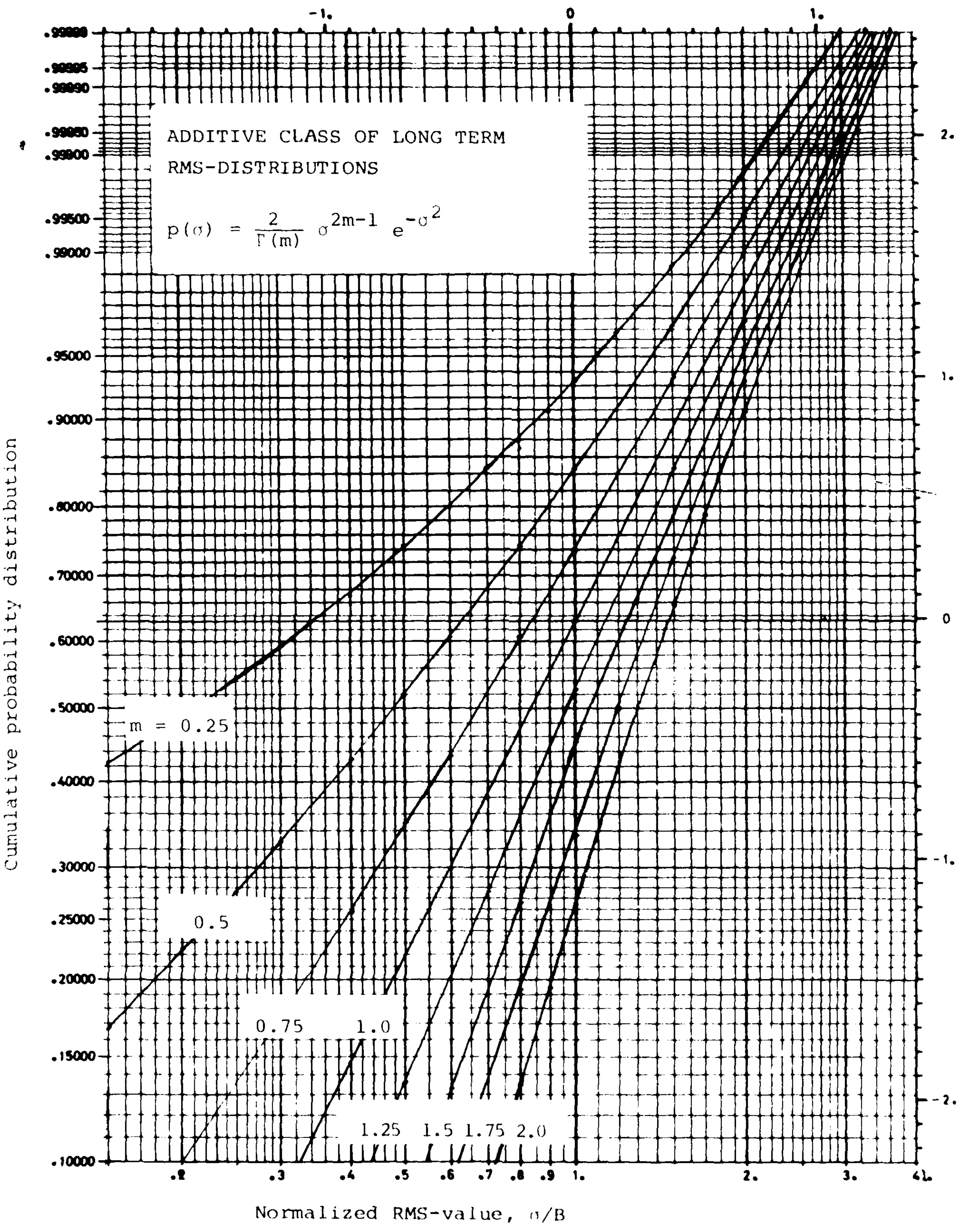

Fig. 9.1.1 Weibull plot of selected members of the additive class of long term probability distributions.

The statistical moments of the distribution (9.1.1) are

$$M_i = E(\sigma^2) = B^i \frac{\Gamma(m+i/2)}{\Gamma(m)} \tag{9.1.5}$$

That is, the parameters m and B can be expressed by moments through

$$m = \frac{M_2{}^2}{M_4 - M_2{}^2} \tag{9.1.6}$$

$$B^2 = M_2/m \tag{9.1.7}$$

In the particular case treated here, the squared springing share $x^2 = (\sigma_s/\sigma)^2$ is Beta-distributed with parameters m and n. The probability density function of x is then

$$p(x) = \frac{2\Gamma(m+n)}{\Gamma(m)\Gamma(n)} x^{2n-1}(1-x^2)^{m-1} \tag{9.1.8}$$

The probability density of the bending share is obtained by interchange of m and n.

9.2 Distribution in general cases

The simple relationship between the individual and the resulting distribution of RMS (9.1.1)-(9.1.4) does not hold in general. Neither does the exact Beta-distribution hold for the squared springing- or bending share.

One may, however, fit the same distributions to the empirical data by methods defined in the sequel:

Variables defined in the interval ($0 - \infty$):

This yields in particular springing, bending and total RMS. Statistical distribution may be approximated with the generalized gamma distribution with parameters (b, g, B) with density

$$f(Z) = \frac{g}{\Gamma(b)B} \left(\frac{Z}{B}\right)^{bg-1} e^{-(Z/B)^g} \tag{9.2.1}$$

The parameters b, k and B may be estimated by the method of moments as defined in /6/ through the following steps:

- The measured values are

$$Z_1 \; Z_2 \; Z_3 \ldots\ldots\ldots Z_N \tag{9.2.2}$$

- Evaluate estimators for the logarithmic mean value, variance and skewness:

$$R = \frac{1}{N} \sum_{i=1}^{N} \ln Z_i \tag{9.2.3}$$

$$V = \frac{1}{N-1} \sum_{i=1}^{N} (\ln Z_i - R)^2 \tag{9.2.4}$$

$$T = S^{-3/2} \frac{1}{(N-1)(N-2)} \sum_{i=1}^{N} (\ln Z_i - R)^3 \tag{9.2.5}$$

- Determine the value of the shape parameter b from the formula

$$-\Psi''(b)/\Psi'(b)^{3/2} = /T/$$

The table in Appendix A may be used.

- Determine the slope parameter k by

$$g = \pm\sqrt{\Psi'(b)/V} \qquad \begin{array}{l} + \text{ for } T < 0 \\ - \text{ for } T > 0 \end{array} \tag{9.2.6}$$

- Determine the scale parameter B through

$$B = \exp\ (R - \Psi(b)/g) \tag{9.2.7}$$

Available computer programs are described in /11/ and /12/. It should be stressed, nowever, that the empirical estimates (9.2.3)-(9.2.5) should, if possible, be calculated directly from the observed values in the sequence (9.2.2). Grouping of data into classes, which is more or less explicitely assumed in the programs has proved to introduce unnecessary inaccuracies, in particular in the determination of the skewness parameter b.

<u>Variables defined in the interval (0,1)</u>

Variables defined in (0,1) are among others:

- The spectral width ε
- The peak-to-zero crossing period io α
- The springing share x
- The bending share x_B
- The fraction of positive maxima, redefined as (2a-1)

The squared values of the variables are also defined in (0,1). The statistical distributions of such variables may be approximated by the Beta-distribution with the density

$$g(Z) = \frac{\Gamma(m+n)}{\Gamma(m)\,\Gamma(n)}\ Z^{n-1}(1-Z)^{m-1} \tag{9.2.8}$$

The two parameters m and n may be determined by the mean value and standard deviation as follows:

- Determine the value $\bar{Z}$ and the variance V through

$$\bar{Z} = \frac{1}{N} \Sigma Z_i \qquad (9.2.9)$$

$$V = \frac{1}{N-1} \Sigma (Z_i - \bar{Z})^2 \qquad (9.2.10)$$

- Determine the parameters n and m through:

$$n = (\bar{Z} - \bar{Z}^2 - \sigma_Z^2)\bar{Z}/V \qquad (9.2.11)$$

$$m = n(1-\bar{Z})/\bar{Z} \qquad (9.2.12)$$

Preferably the squared variables ε^2, x^2 etc. should be matched to the Beta-distribution, because then the distributions of $\alpha^2 = 1 - \varepsilon^2$ and $x_B^2 = 1 - x^2$ are simultaneously determined. The probability distribution of the variable themselves, i.e. ε or x are then given by the function (9.1.8).

10. LONG TERM DISTRIBUTION OF POSITIVE MAXIMA

10.1 Proposed procedure

Suggest that the distribution of local maxima under stationary conditions can be described by a general gamma distribution with parameters (a, h, A). By section 3.3 this distribution should approximate the truncated Rice distribution, giving in particular

$$\begin{aligned} a &= \text{fraction of positive maxima in interval } (0.5,\ 1) \\ h &= 2 \qquad\qquad (10.1.1) \\ A &= \sqrt{2}\sigma \end{aligned}$$

The scale parameter A is distributed in the long run according to a general gamma function with parameters (b, g, B).

Neglecting period fluctuations for the moment, the long term distribution of local maxima is

$$f_S(S) = \int_0^\infty f(a,\ h,\ A;\ S)\ f\ (b,\ g,\ B;\ A)\ dA \qquad (10.1.2)$$

This distribution can be approximated by a general gamma distribution with parameters (d, k, D) by a method which gives correct logarithmic moments up to the third order.

Transforming (10.1.2) to lnS, one may find the moment generating functions of the distributions on each side of the equality sign. Equating the moment generating functions gives

$$\Phi(u) = E(e^{us}) = D^u \frac{\Gamma(d+u/k)}{\Gamma(d)} = B^u \frac{\Gamma(a+u/h)\Gamma(b+u/g)}{\Gamma(a)\Gamma(b)} \qquad (10.1.3)$$

Hence the cumulant generating function is

$$\Theta(u) = \ln \Phi(u) \tag{10.1.4}$$

From this function the cumulants κ_n of general order r can be derived by

$$n = \left.\frac{d\Theta}{du}\right|_{u=o} \tag{10.1.5}$$

Comparing the first three cumulants of (10.1.3) gives the three equations for determination of the gamma parameters d,k and A.

$$\kappa_1 = \ln D + \Psi(d)/k = \ln B + \Psi(a)/h + \Psi(b)/g \tag{10.1.6}$$

$$\kappa_2 = \Psi^1(d)/k^2 = \Psi^1(a)/h^2 + \Psi^1(b)/g^2 \tag{10.1.7}$$

$$\kappa_3 = \Psi^{11}(d)/k^3 = \Psi^{11}(a)/h^3 + \Psi^{11}(b)/g^3 \tag{10.1.8}$$

Hence the skewness coefficient on each side is

$$\frac{\kappa_3}{\kappa_2{}^{3/2}} = \frac{\Psi^{11}(d)}{\Psi^1(d)^{3/2}} = \frac{\Psi^{11}(a)/h^3 + \Psi^{11}(b)/g^3}{\left[\Psi^1(a)/h^2 + \Psi^1(b)/g^2\right]^{3/2}} \tag{10.1.9}$$

The right side is known, and the middle term is only a function of d. Hence d can be evaluated by the table in Appendix A..

Once d is known, k may be evaluated from the second cumulant identity (10.1.7)

$$k = \left[\frac{\Psi^1(d)}{\Psi^1(a)/h^2 + \Psi^1(b)/g^2}\right]^{1/2} \tag{10.1.10}$$

and finally from the first cumulant (10.1.6) one finds

$$D = B \exp\left\{\Psi(a)/h + \Psi(b)/g - \Psi(d)/k\right\} \tag{10.1.11}$$

In the present particular case of Rice-distributed short term maxima we have $h = 2$.
If the long term distribution of the RMS value σ rather than of $A=\sqrt{2}\sigma$ is known, B should be set equal to 2 x the scale parameter of the long term distribution of σ.

To get an idea about the validity of the procedure proposed in the last section, one may consider the particular case of narrow banded stresses (Rayleigh distribution) where the $A = \sqrt{2}$ RMS is Weibull distributed in the l ng run.
In this case we have

$$a = 1$$
$$h = 2$$
$$b = 1$$
$$\Psi(1) = 0.57721$$
$$\Psi^{1}(1) = 1.64493$$
$$\Psi^{11}(1) = 2.40411$$

When the Weibull parameters k and B of the long term distribution of $\sqrt{2}$ RMS are known, the gamma parameters d,k and D of the amplitude distribution may be determined from (10.1.9)-(10.1.11).

Some corresponding values are given in Table 10.2.1.

g	d	k	(D/B)
0.0	0.65	0	0.749
0.5	1.16	0.436	0.516
1.0	1.50	0.674	0.399
1.5	1.79	0.806	0.361
2.0	1.89	0.918	0.384
2.5	1.83	1.034	0.442
3.0	1.72	1.147	0.508
3.5	1.60	1.254	0.575
4.0	1.50	1.349	0.631
4.5	1.42	1.429	0.678
5.0	1.35	1.503	0.720
5.5	1.30	1.561	0.752
6.0	1.26	1.610	0.778

Table 10.2.1. Parameters of long term distribution of amplitudes by Rayleigh distributed short amplitudes and Weibull distributed $\sqrt{2}$ RMS.

Corresponding values can also be determined from Fig.10.2.1.

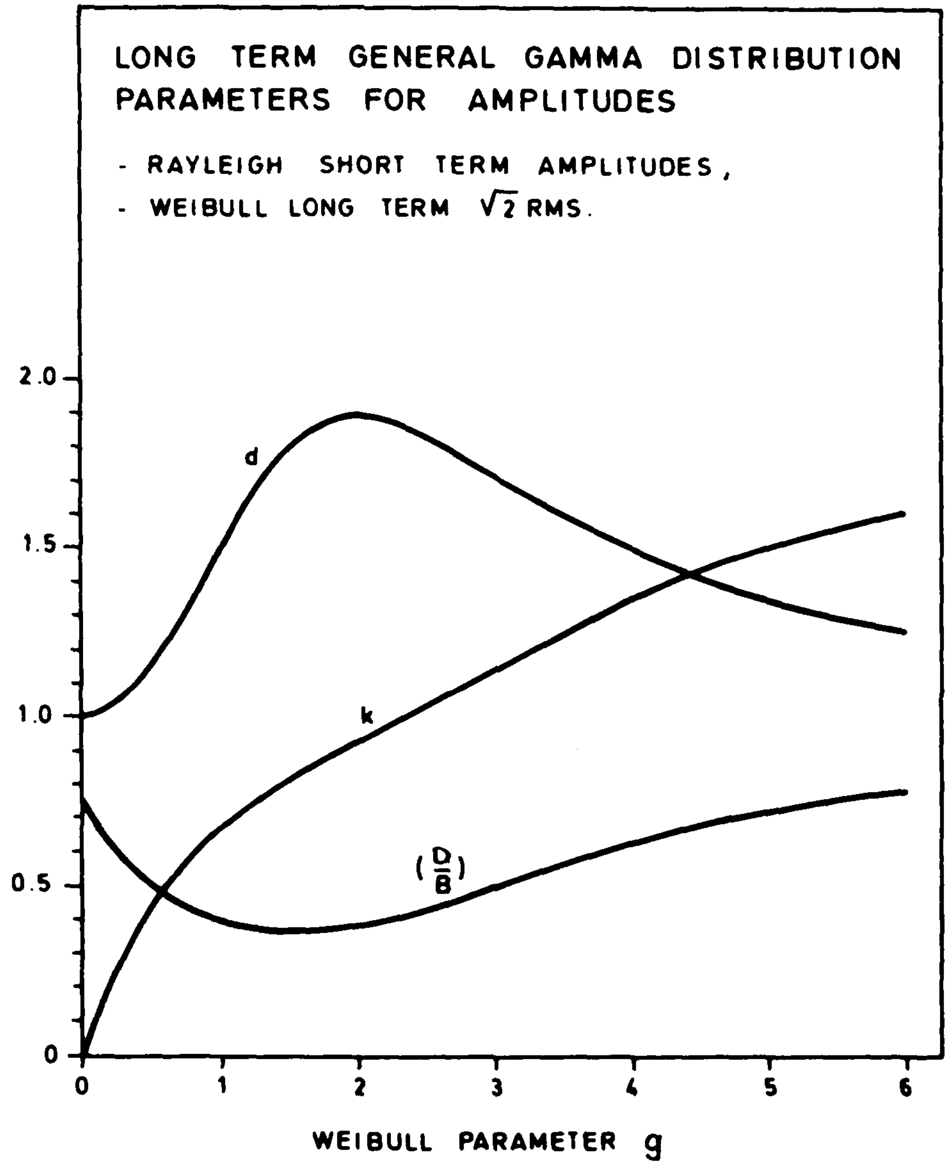

Fig.10.2.1. Graph of gamma parameters for long term stress amplitudes (d,K,D) by Rayleigh short term amplitudes ($1,2,\sqrt{2}\sigma$) and Weibull long term $\sqrt{2}\sigma$, that is ($1,g,B$).

Comparison may be made by a procedure reported by Nordenstrøm /13/ where the objective was to fit a Weibull distribution to (10.1.2) which should give good accuracy for large stresses.

Some corresponding values for d,k and D obtained by the two methods are listed in Table 10.2.2.

	Nordenstrøm's values			Present method		
g	d	k	D/B	d	k	D/B
0.5	1.0	.428	.560	1.16	.436	.516
1.0	1.0	.722	.611	1.50	.674	.399
2.0	1.0	1.086	.690	1.89	.918	.384
4.0	1.0	1.444	.782	1.50	1.349	.631
6.0	1.0	1.614	.834	1.26	1.610	.778
∞	1.0	2.0	1.0	1.0	2.0	1.0

Table 10.2.2. Some corresponding amplitude distribution parameters obtained by different methods.

Weibull plots of the distribution obtained for g=1.0 and 6.0 are shown in Fig.10.2.2. The corresponding distributions are seen to coincide for large stresses where Nordenstrøms values are most correct. This investigation gives some confidence to the present procedure, at least for low spectral width.

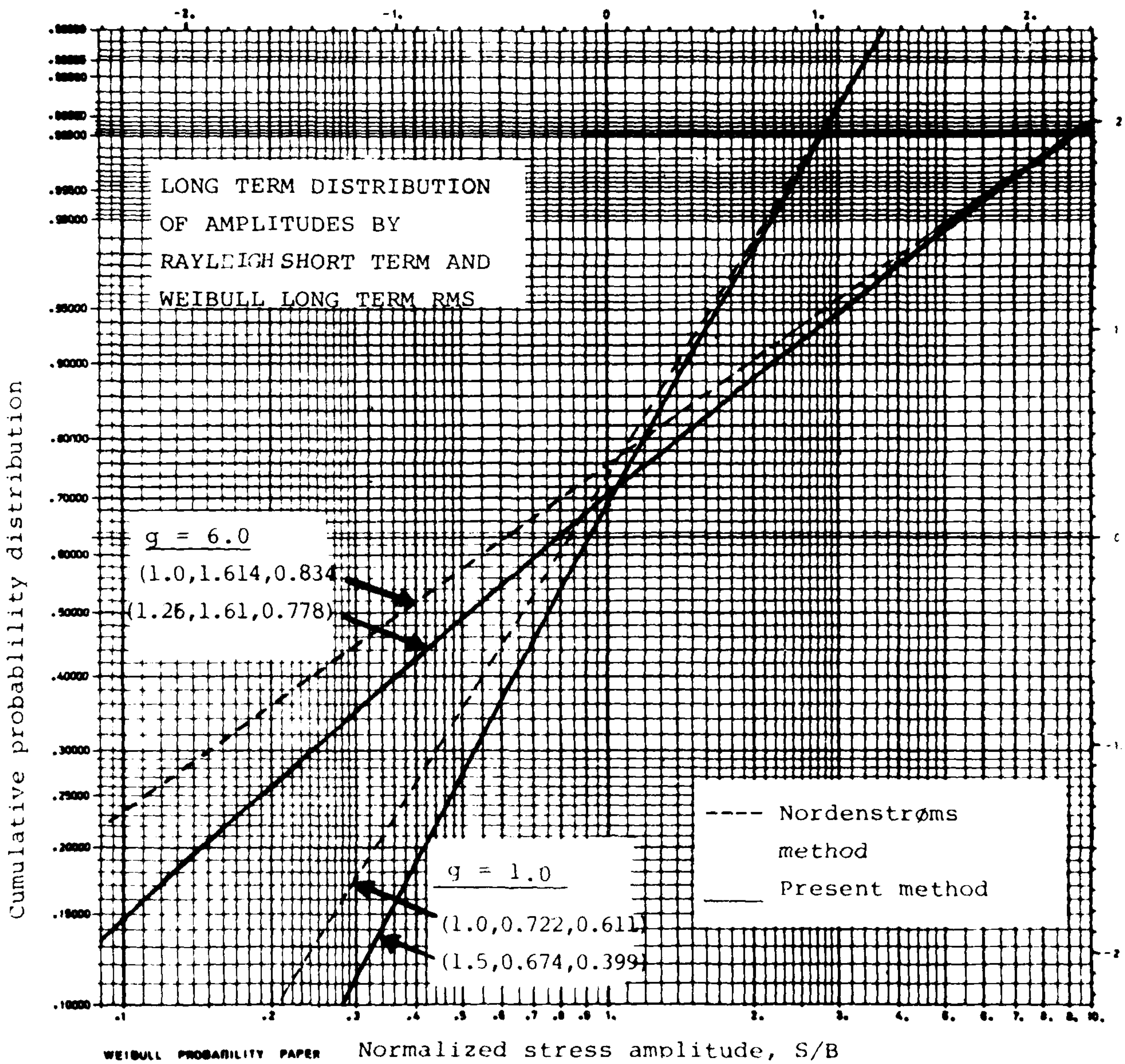

Fig.1o.2.2. Long term Cumulative probability distribution of narrow banded stress amplitudes by Weibull long term distribution for $\sqrt{2}\sigma$ Comparison between the present method and data from Nordenstrøm /13/. g is the slope parameter for the Weibull distribution of $\sqrt{2}\sigma$.

11. CONCLUDING REMARKS.

The validity of the way of establishing the long term stress distribution outlined in chapter 10 should be more thoroughly studied, in particular for wide band stresses.

The long term parameters (d,k,D) for half-normal short term stress distribution and Weibull long term D-distribution are given in Table 11.1 and Fig.11.1 analogous to the representation in Section 10.2. However, alternative data for comparison are not imediately available.

When the long term statistical distribution of local maxima is established in terms of the probability distribution P(d,k,D;S) the characteristic long term extreme can be established as the $(1-N_p^+)$ fractile, This may always be solved numerically, for instance by /11/ which solves this by Wegstein iteration.

One may also evaluate a characteristic extreme value by the asymptotic expression

$$S_c = D\left\{\ln\left(\frac{d}{\Gamma(d)}\right)N_p^+ + (d-1/k)\ln\left[\ln\left(\frac{d}{\Gamma(d)}\right)N_p^+\right]\right\}^{1/k} \qquad (11.1.1)$$

The probability distribution of the extreme value is given as

$$P(z) = P(d,k,D;S)^{N_p^+} \qquad (11.1.2)$$

analogous to (5.1.1), and may be discussed along much the same lines as in the stationary case Chapter 5.

The influence of the changes in period should be investigated.

g	d	k	A/B
.0	1.00	0	.374
.1	1.02	.0982	.299
.2	1.08	.186	.237
.3	1.18	.258	.185
.4	1.32	.311	.142
.5	1.49	.347	.1092
.6	1.68	.374	.0874
.7	1.90	.387	.0655
.8	2.13	.396	.0511
.9	2.36	.401	.0408
1.0	2.58	.405	.0337
1.5	3.35	.421	.0209
2.0	3.53	.446	.0231
2.5	3.46	.473	.0297
3.0	3.35	.495	.0369
3.5	3.24	.514	.0442
4.0	3.15	.528	.0507
4.5	3.08	.540	.0563
5.0	3.03	.548	.0607
5.5	2.98	.556	.0651
6.0	2.95	.561	.0681

Table 11.1 Parameters of long term distribution of positive maxima by broad band signal ($\varepsilon=1$) and Weibull distributed $\sqrt{2}$ RMS.

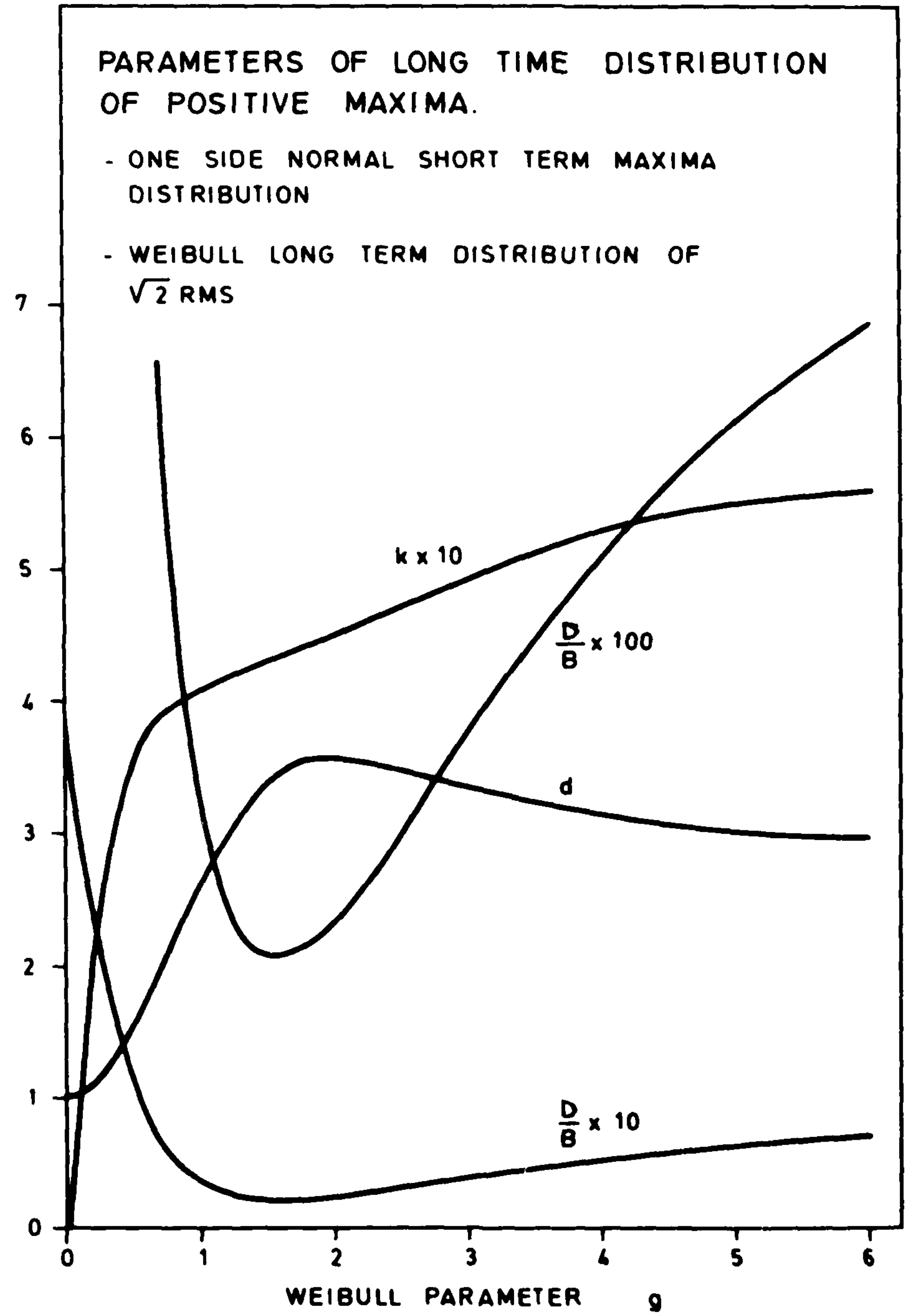

Fig. 11.1 Graph of gamma parameters for long term stress amplitudes (d,k,D) by one-sided normal short term distribution ($\frac{1}{2}$,2,$\sqrt{2}\sigma$ with parameters (1,g,B).

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APPENDIX A

TABLE OF POLY-GAMMA AND RELATED FUNCTIONS.

Following quantities are tabelled:

X	-	argument of functions
GAMMA	-	$\Gamma(x)$
PS	-	$\psi(x)$
PS1	-	$\psi'(x)$
PS2	-	$\psi''(x)$
PS/VPS1	-	$\psi(x)/\sqrt{\psi'(x)}$
VPS	-	$\sqrt{\psi'(x)}$
PS2/PS1xx3/2	-	$\psi''(x)/\psi'(x)^{3/2}$

X	GAMMA ,	PS	PS1	PS2	PS/VPS1	VPS1	PS2/PS1**3/2
.01	99.4325	-100.56	10001.6	-2000006.	-1.0055	100.008	-1.9995
.02	49.4422	-50.54	2501.5	-250002.	-1.0106	50.016	-1.9981
.03	32.7[illegible]50	-33.[illegible]	1112.7	-74076.	-1.0151	33.357	-1.9958
.04	24.4610	-25.51	626.6	-31252.	-1.01[illegible]5	25.031	-1.9927
.05	19.4701	-20.50	401.5	-16002.	-1.0229	20.038	-1.9888
.06	16.1457	-17.15	279.3	-9261.	-1.0262	16.712	-1.9842
.07	13.7736	-14.75	205.6	-5833.	-1.0290	14.338	-1.9790
.08	11.[illegible]66	-12.[illegible]5	157.7	-3[illegible].	-1.0314	12.559	-1.9731
.09	10.6162	-11.55	124.9	-2745.	-1.0334	11.176	-1.9666
.10	9.5135	-10.42	101.4	-2002.	-1.0350	10.071	-1.9596
.11	8.6127	-9.50	84.1	-1504.	-1.0362	9.168	-1.9521
.12	7.8632	-8.73	70.8	-1159.	-1.0371	8.417	-1.9441
.13	7.2302	-8.07	60.6	-912.	-1.0376	7.781	-1.9357
.14	6.6887	-7.51	52.4	-731.	-1.0377	7.238	-1.9270
.15	6.2203	-7.0210	45.7900	-594.2466	-1.0376	6.7668	-1.9178
.16	5.8113	-6.5910	40.3717	-4[illegible].8977	-1.0371	6.3554	-1.9084
.17	5.4512	-6.2101	35.9153	-408.6635	-1.0362	5.9929	-1.8987
.18	5.1319	-5.8702	32.1618	-344.4806	-1.0351	5.6711	-1.8887
.19	4.8469	-5.5649	28.9832	-293.0787	-1.0337	5.3836	-1.8784
.20	4.5908	-5.2830	26.2674	-251.4781	-1.0320	5.1252	-1.8680
.21	4.3599	-5.03[illegible]3	23.9285	-217.4055	-1.0300	4.8917	-1.8574
.22	4.1505	-4.8094	21.8996	-189.2439	-1.0277	4.6797	-1.8466
.23	3.95[illegible]8	-4.5995	20.1280	-165.7642	-1.0252	4.4864	-1.8356
.24	3.7855	-4.4062	18.5719	-146.0319	-1.0224	4.3095	-1.8246
.25	3.6256	-4.2275	17.1973	-128.3277	-1.0194	4.1470	-1.8134
.26	3.47[illegible]5	-4.0617	15.9771	-115.0919	-1.0162	3.9971	-1.8022
.27	3.3426	-3.9075	14.8887	-102.8943	-1.0127	3.8586	-1.7909
.28	3.2169	-3.7635	13.9135	-92.3558	-1.0090	3.7301	-1.7795
.29	3.1001	-3.6289	13.0370	-83.2270	-1.0050	3.6107	-1.7681
.30	2.9916	-3.5025	12.2454	-75.2725	-1.0009	3.4993	-1.7566
.31	2.8903	-3.3837	11.5282	-68.3092	-.9966	3.3953	-1.7452
.32	2.795[illegible]	-3.2717	10.8764	-62.1870	-.9921	3.2979	-1.7337
.33	2.7072	-3.1660	10.2321	-56.7824	-.9873	3.2066	-1.7222
.34	2.6242	-3.0659	9.7387	-51.9931	-.9825	3.1207	-1.7108
.35	2.5461	-2.9711	9.2405	-47.7338	-.9774	3.0398	-1.6994
.36	2.4727	-2.8810	8.7825	-43.9329	-.9721	2.9635	-1.6880
.37	2.4035	-2.7953	8.3605	-40.5303	-.9667	2.8914	-1.6766
.38	2.3383	-2.7137	7.9707	-37.4750	-.9612	2.8232	-1.6653
.39	2.2765	-2.6358	7.6100	-34.7236	-.9555	2.7586	-1.6541
.40	2.2182	-2.5614	7.2754	-32.2341	-.9496	2.6973	-1.6429
.41	2.162[illegible]	-2.4902	6.9644	-29.9893	-.9436	2.6390	-1.6317
.42	2.1104	-2.4220	6.6749	-27.9496	-.9375	2.5836	-1.6207
.43	2.0605	-2.3566	6.4047	-26.0917	-.9312	2.5303	-1.6097
.44	2.0132	-2.2939	6.1525	-24.3986	-.9248	2.4804	-1.5988
.45	1.9681	-2.2335	5.9164	-22.8517	-.9183	2.4324	-1.5880
.46	1.9252	-2.1755	5.6950	-21.4355	-.9116	2.3864	-1.5772
.47	1.8843	-2.1196	5.4873	-20.1362	-.9048	2.3425	-1.5666
.48	1.8453	-2.0657	5.2919	-18.9422	-.8980	2.3004	-1.5560
.49	1.8081	-2.0137	5.1081	-17.8429	-.8910	2.2601	-1.5455

X	GAMMA	PS	PS1	PS2	PS/VPS1	VPS1	PS2/PS1**3/2
.50	1.7725	-1.9635	4.9344	-16.8238	-.8839	2.2214	-1.5351
.51	1.7384	-1.9130	4.7713	-15.9920	-.8767	2.1843	-1.5249
.52	1.7053	-1.8631	4.6167	-15.0252	-.8694	2.1497	-1.5147
.53	1.6747	-1.8226	4.4705	-14.2220	-.8620	2.1144	-1.5046
.54	1.6442	-1.7786	4.3321	-13.4764	-.8545	2.0814	-1.4946
.55	1.6161	-1.7360	4.2008	-12.7835	-.8470	2.0496	-1.4847
.56	1.5886	-1.6946	4.0763	-12.1336	-.8393	2.0190	-1.4749
.57	1.5623	-1.6544	3.9572	-11.5376	-.8316	1.9895	-1.4653
.58	1.5369	-1.6154	3.8454	-10.9763	-.8238	1.9610	-1.4557
.59	1.5126	-1.5775	3.7383	-10.4523	-.8159	1.9335	-1.4462
.60	1.4892	-1.5406	3.6362	-9.9628	-.8079	1.9069	-1.4368
.61	1.4667	-1.5047	3.5389	-9.5039	-.7999	1.8812	-1.4276
.62	1.4450	-1.4698	3.4460	-9.0737	-.7918	1.8564	-1.4184
.63	1.4242	-1.4353	3.3573	-8.6699	-.7836	1.8323	-1.4094
.64	1.4041	-1.4027	3.2726	-8.2905	-.7754	1.8090	-1.4004
.65	1.3848	-1.3703	3.1915	-7.9337	-.7671	1.7865	-1.3915
.66	1.3662	-1.3388	3.1139	-7.5979	-.7587	1.7646	-1.3828
.67	1.3482	-1.3081	3.0394	-7.2814	-.7503	1.7434	-1.3741
.68	1.3309	-1.2780	2.9631	-6.9829	-.7418	1.7228	-1.3656
.69	1.3142	-1.2487	2.8997	-6.7012	-.7333	1.7029	-1.3571
.70	1.2981	-1.2200	2.8340	-6.4350	-.7247	1.6835	-1.3488
.71	1.2825	-1.1920	2.7710	-6.1833	-.7161	1.6646	-1.3405
.72	1.2675	-1.1646	2.7103	-5.9450	-.7074	1.6463	-1.3324
.73	1.2530	-1.1378	2.6520	-5.7194	-.6987	1.6285	-1.3243
.74	1.2390	-1.1115	2.5959	-5.5055	-.6899	1.6112	-1.3163
.75	1.2254	-1.0859	2.5419	-5.3026	-.6811	1.5943	-1.3085
.76	1.2123	-1.0607	2.4899	-5.1100	-.6722	1.5779	-1.3007
.77	1.1997	-1.0361	2.4396	-4.9271	-.6633	1.5619	-1.2930
.78	1.1875	-1.0119	2.3913	-4.7531	-.6544	1.5464	-1.2854
.79	1.1757	-.9882	2.3446	-4.5876	-.6454	1.5312	-1.2779
.80	1.1642	-.9650	2.2995	-4.4301	-.6364	1.5164	-1.2705
.81	1.1532	-.9422	2.2559	-4.2801	-.6273	1.5020	-1.2632
.82	1.1425	-.9199	2.2138	-4.1370	-.6182	1.4879	-1.2559
.83	1.1322	-.8980	2.1732	-4.0006	-.6091	1.4742	-1.2488
.84	1.1222	-.8764	2.1338	-3.8705	-.6000	1.4608	-1.2417
.85	1.1125	-.8553	2.0957	-3.7462	-.5908	1.4477	-1.2348
.86	1.1031	-.8345	2.0589	-3.6274	-.5816	1.4349	-1.2279
.87	1.0941	-.8141	2.0232	-3.5138	-.5723	1.4224	-1.2211
.88	1.0853	-.7940	1.9886	-3.4052	-.5631	1.4102	-1.2143
.89	1.0768	-.7743	1.9551	-3.3013	-.5538	1.3982	-1.2077
.90	1.0686	-.7549	1.9225	-3.2018	-.5445	1.3866	-1.2011
.91	1.0607	-.7359	1.8910	-3.1065	-.5351	1.3751	-1.1946
.92	1.0530	-.7171	1.8604	-3.0151	-.5257	1.3640	-1.1882
.93	1.0456	-.6986	1.8307	-2.9274	-.5164	1.3530	-1.1819
.94	1.0384	-.6805	1.8019	-2.8434	-.5069	1.3423	-1.1756
.95	1.0315	-.6626	1.7738	-2.7627	-.4975	1.3318	-1.1694
.96	1.0247	-.6450	1.7466	-2.6852	-.4881	1.3216	-1.1633
.97	1.0182	-.6277	1.7201	-2.6107	-.4786	1.3115	-1.1573
.98	1.0119	-.6106	1.6943	-2.5391	-.4691	1.3017	-1.1513
.99	1.0059	-.5938	1.6693	-2.4703	-.4596	1.2920	-1.1454

X	GAMMA	PS	PS1	PS2	PS/VPS1	VPS1	PS2/PS1**3/2
1.00	1.0000	-.5772	1.6449	-2.4041	-.4501	1.2825	-1.1395
1.01	.9943	-.5608	1.6212	-2.3404	-.4405	1.2733	-1.1339
1.02	[illegible]	-.5448	1.5981	-2.2791	-.4309	1.2642	-1.1281
1.03	.9835	-.5290	1.5756	-2.2200	-.4214	1.2552	-1.1225
1.04	.9784	-.5133	1.5537	-2.1630	-.4113	1.2465	-1.1169
1.05	.9735	-.4978	1.5324	-2.1082	-.4022	1.2379	-1.1114
1.06	.9687	-.4826	1.5115	-2.0552	-.3926	1.2294	-1.1059
1.07	.9642	-.4676	1.4912	-2.0042	-.3829	1.2212	-1.1006
1.08	.9597	-.4528	1.4715	-1.9549	-.3733	1.2130	-1.0952
1.09	.9555	-.4382	1.4521	-1.9074	-.3636	1.2050	-1.0900
1.10	.9514	-.4239	1.4333	-1.8615	-.3540	1.1972	-1.0848
1.11	.9474	-.4095	1.4149	-1.8171	-.3443	1.1895	-1.0797
1.12	.9436	-.3955	1.3970	-1.7742	-.3346	1.1819	-1.0746
1.13	.9399	-.3816	1.3794	-1.7328	-.3249	1.1745	-1.0696
1.14	.9364	-.3679	1.3623	-1.6927	-.3152	1.1672	-1.0646
1.15	.9330	-.3543	1.3456	-1.6540	-.3055	1.1600	-1.0597
1.16	.9298	-.3410	1.3292	-1.6165	-.2957	1.1529	-1.0548
1.17	.9267	-.3277	1.3132	-1.5802	-.2860	1.1460	-1.0500
1.18	.9237	-.3147	1.2976	-1.5450	-.2763	1.1391	-1.0453
1.19	.9209	-.3018	1.2823	-1.5110	-.2665	1.1324	-1.0406
1.20	.9182	-.2890	1.2674	-1.4780	-.2567	1.1258	-1.0359
1.21	.9156	-.2764	1.2528	-1.4461	-.2470	1.1193	-1.0313
1.22	.9131	-.2640	1.2385	-1.4151	-.2372	1.1129	-1.0268
1.23	.9108	-.2517	1.2245	-1.3851	-.2274	1.1065	-1.0223
1.24	.9085	-.2395	1.2107	-1.3560	-.2177	1.1003	-1.0178
1.25	.9064	-.2275	1.1973	-1.3277	-.2079	1.0942	-1.0134
1.26	.9044	-.2155	1.1842	-1.3003	-.1981	1.0882	-1.0091
1.27	.9025	-.2038	1.1713	-1.2737	-.1883	1.0823	-1.0049
1.28	.9007	-.1921	1.1587	-1.2479	-.1785	1.0764	-1.0005
1.29	.8990	-.1806	1.1464	-1.2228	-.1687	1.0707	-.9963
1.30	.8975	-.1692	1.1343	-1.1985	-.1589	1.0650	-.9921
1.31	.8960	-.1579	1.1224	-1.1749	-.1491	1.0594	-.9880
1.32	.8946	-.1467	1.1108	-1.1518	-.1392	1.0539	-.9839
1.33	.8934	-.1357	1.0994	-1.1294	-.1294	1.0485	-.9799
1.34	.8922	-.1248	1.0882	-1.1077	-.1196	1.0432	-.9758
1.35	.8912	-.1139	1.0772	-1.0866	-.1098	1.0379	-.9719
1.36	.8902	-.1032	1.0664	-1.0660	-.0999	1.0327	-.9680
1.37	.8893	-.0926	1.0559	-1.0460	-.0901	1.0276	-.9641
1.38	.8885	-.0821	1.0455	-1.0265	-.0803	1.0225	-.9602
1.39	.8879	-.0717	1.0353	-1.0076	-.0705	1.0175	-.9564
1.40	.8873	-.0614	1.0254	-.9891	-.0606	1.0126	-.9527
1.41	.8868	-.0512	1.0156	-.9712	-.0508	1.0077	[illegible]
1.42	.8864	-.0411	1.0059	-.9537	-.0410	1.0030	[illegible]
1.43	.8860	-.0311	[illegible]	-.9366	-.0311	[illegible]	[illegible]
1.44	.8858	-.0211	.9872	-.9200	-.0213	.9936	[illegible]
1.45	.8857	-.0113	[illegible]	[illegible]	-.0114	[illegible]	[illegible]
1.46	.8856	-.0016	.9691	[illegible]	[illegible]	[illegible]	[illegible]
1.47	[illegible]	.0081	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
1.48	[illegible]	.0176	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
1.49	[illegible]	.0271	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]

AD-A111 186 NORSKE VERITAS OSLO F/G 13/10

STATISTICAL DESCRIPTION OF WAVE INDUCED VIBRATORY STRESSES IN S--ETC(U)

DEC 80 S GRAN DTCG23-80-C-20007

UNCLASSIFIED 80-1171 USCG-M-2-81 NL

2 OF 2

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DTIC

X	GAMMA	PS	PS1	PS2	PS/VPS1	VPS1	PS2/PS1**3/2
1.50	.8862	.0365	.9349	-.8299	.0377	.9669	-.9170
1.51	.8866	.0457	.9266	-.8149	.0476	.9626	-.9136
1.52	.8870	.0550	.9185	-.8013	.0574	.9584	-.9103
1.53	.8876	.0642	.9106	-.7879	.0672	.9542	-.9070
1.54	.8882	.0732	.9027	-.7751	.0771	.9501	-.9037
1.55	.8889	.0822	.8951	-.7625	.0869	.9461	-.9004
1.56	.8896	.0911	.8875	-.7501	.0967	.9421	-.8972
1.57	.8905	.1000	.8801	-.7381	.1066	.9381	-.8940
1.58	.8914	.1087	.8727	-.7263	.1164	.9342	-.8908
1.59	.8924	.1174	.8655	-.7148	.1262	.9303	-.8877
1.60	.8935	.1260	.8584	-.7036	.1360	.9265	-.8846
1.61	.8947	.1346	.8515	-.6926	.1459	.9227	-.8815
1.62	.8959	.1431	.8446	-.6817	.1557	.9190	-.8785
1.63	.8972	.1515	.8379	-.6714	.1655	.9153	-.8755
1.64	.8986	.1598	.8312	-.6611	.1753	.9117	-.8725
1.65	.9001	.1681	.8246	-.6511	.1851	.9081	-.8695
1.66	.9017	.1763	.8181	-.6413	.1949	.9045	-.8666
1.67	.9033	.1845	.8118	-.6317	.2047	.9010	-.8637
1.68	.9050	.1926	.8055	-.6223	.2146	.8975	-.8608
1.69	.9068	.2006	.7993	-.6131	.2244	.8940	-.8579
1.70	.9086	.2085	.7932	-.6041	.2342	.8906	-.8551
1.71	.9106	.2164	.7872	-.5953	.2440	.8873	-.8523
1.72	.9126	.2243	.7813	-.5867	.2537	.8839	-.8495
1.73	.9147	.2321	.7755	-.5782	.2635	.8806	-.8467
1.74	.9168	.2398	.7698	-.5700	.2733	.8774	-.8440
1.75	.9191	.2475	.7641	-.5619	.2831	.8741	-.8413
1.76	.9214	.2551	.7585	-.5540	.2929	.8709	-.8386
1.77	.9238	.2626	.7530	-.5462	.3027	.8678	-.8359
1.78	.9262	.2701	.7476	-.5386	.3124	.8646	-.8332
1.79	.9288	.2776	.7422	-.5312	.3222	.8615	-.8306
1.80	.9314	.2850	.7370	-.5239	.3320	.8585	-.8280
1.81	.9341	.2923	.7318	-.5167	.3417	.8554	-.8254
1.82	.9369	.2996	.7266	-.5097	.3515	.8524	-.8229
1.83	.9397	.3069	.7216	-.5028	.3613	.8495	-.8203
1.84	.9426	.3141	.7166	-.4961	.3710	.8465	-.8178
1.85	.9456	.3212	.7117	-.4895	.3808	.8436	-.8153
1.86	.9487	.3283	.7068	-.4830	.3905	.8407	-.8129
1.87	.9518	.3353	.7020	-.4767	.4002	.8379	-.8104
1.88	.9551	.3423	.6973	-.4704	.4100	.8350	-.8080
1.89	.9584	.3493	.6926	-.4643	.4197	.8322	-.8056
1.90	.9618	.3562	.6880	-.4583	.4294	.8294	-.8032
1.91	.9652	.3630	.6834	-.4524	.4391	.8267	-.8008
1.92	.9688	.3699	.6789	-.4467	.4489	.8240	-.7984
1.93	.9724	.3765	.6745	-.4410	.4586	.8213	-.7961
1.94	.9761	.3833	.6701	-.4354	.4683	.8186	-.7938
1.95	.9799	.3900	.6658	-.4300	.4780	.8160	-.7915
1.96	.9837	.3967	.6615	-.4246	.4877	.8133	-.7892
1.97	.9877	.4033	.6573	-.4193	.4974	.8107	-.7869
1.98	.9917	.4098	.6531	-.4142	.5071	.8082	-.7847
1.99	.9958	.4163	.6490	-.4091	.5168	.8056	-.7825

X	GAMMA	PS	PS1	PS2	PS/VPS1	VPS1	PS2/PS1**3/2
2.00	1.0000	.422[illegible]	.644[illegible]	-.4041	.5265	.8031	-.7802
2.01	1.0043	.42[illegible]	[illegible]	-.39[illegible]2	.5361	.8006	-.7780
2.02	1.0[illegible]6	.4[illegible]	[illegible]	-.3[illegible]44	.545[illegible]	.7981	-.7759
2.03	1.0131	.4420	.6330	-.39[illegible]7	.5555	.7956	-.7737
2.04	1.0176	.44[illegible]3	.62[illegible]	-.3850	.5651	.7932	-.7716
2.05	1.022[illegible]	.4545	.6[illegible]53	-.3805	.5743	.7908	-.7694
2.06	1.02[illegible]	.4608	.[illegible]215	-.3760	.5844	.7884	-.7673
2.07	1.0316	.4670	.617[illegible]	-.3716	.5941	.7860	-.7652
2.08	1.0365	.4731	.[illegible]141	-.3673	.6037	.7837	-.7631
2.09	1.0415	.4792	.6105	-.3630	.6134	.7813	-.7611
2.10	1.0465	.4853	.6069	-.3588	.6230	.7790	-.7590
2.11	1.0516	.4[illegible]14	.6033	-.3547	.6326	.7767	-.7570
2.12	1.0569	.4[illegible]74	.59[illegible]	-.3507	.6423	.7744	-.7550
2.13	1.0621	.5034	.5963	-.3467	.6519	.7722	-.7530
2.14	1.0675	.5093	.5928	-.3428	.6615	.7700	-.7510
2.15	1.0730	.5152	.5894	-.3389	.6711	.7677	-.7490
2.16	1.07[illegible]6	.5211	.5860	-.3352	.6807	.7655	-.7470
2.17	1.0[illegible]42	.5270	.5827	-.3314	.6903	.7634	-.7451
2.18	1.0[illegible]00	.5328	.5794	-.3278	.6999	.7612	-.7432
2.19	1.0959	.5385	.5762	-.3242	.7095	.7591	-.7413
2.20	1.1018	.5443	.5729	-.3206	.7191	.7569	-.7393
2.21	1.1078	.5500	.5697	-.3171	.7287	.7548	-.7375
2.22	1.1140	.5557	.5665	-.3137	.7382	.7527	-.7356
2.23	1.1202	.5613	.5635	-.3103	.7478	.7506	-.7337
2.24	1.1266	.5670	.5604	-.3070	.7574	.7486	-.7319
2.25	1.1330	.5725	.5573	-.3037	.7669	.7465	-.7300
2.26	1.1395	.5781	.5543	-.3005	.7765	.7445	-.7282
2.27	1.1462	.5836	.5513	-.2974	.7860	.7425	-.7264
2.28	1.1529	.5891	.5484	-.2942	.7956	.7405	-.7246
2.29	1.1598	.5946	.5454	-.2912	.8051	.7385	-.7228
2.30	1.1667	.6000	.5425	-.2881	.8146	.7366	-.7210
2.31	1.1739	.6055	.5397	-.2852	.8242	.7346	-.7193
2.32	1.1[illegible]0[illegible]	.6108	.5368	-.2822	.8337	.7327	-.7175
2.33	1.1882	.6162	.5340	-.2793	.8432	.7308	-.7158
2.34	1.1956	.6215	.5312	-.2765	.8527	.7289	-.7140
2.35	1.2031	.6268	.5285	-.2737	.8622	.7270	-.7123
2.36	1.2106	.6321	.5258	-.2709	.8717	.7251	-.7106
2.37	1.2184	.6373	.5231	-.2682	.8812	.7232	-.7089
2.38	1.2262	.6425	.5204	-.2655	.8907	.7214	-.7072
2.39	1.2341	.6477	.5178	-.2629	.9002	.7196	-.7056
2.40	1.2422	.6529	.5152	-.2603	.9097	.7177	-.7039
2.41	1.2503	.6580	.5126	-.2577	.9191	.7159	-.7023
2.42	1.2[illegible]6	.6632	.5100	-.2552	.9286	.7141	-.7006
2.43	1.2670	.6682	.5075	-.2527	.9381	.7124	-.6990
2.44	1.2756	.6733	.5049	-.2502	.9475	.7106	-.6974
2.45	1.2[illegible]42	.6783	.5025	-.2478	.9570	.7088	-.6958
2.46	1.2[illegible]30	.[illegible]34	.5000	-.2454	.9664	.7071	-.6942
2.47	1.301[illegible]	.6[illegible]33	.4[illegible]75	-.2431	.9759	.7054	-.6926
2.48	1.310[illegible]	.6[illegible]33	.4[illegible]51	-.2407	.9853	.7037	-.6910
2.49	1.3201	.6982	.4927	-.2385	.9947	.7019	-.6894

X	GAMMA	PS	PS1	PS2	PS/VPS1	VPS1	PS2/PS1**3/2
2.50	1.32[illegible]3	.7032	.4904	-.2362	1.0041	.7003	-.6979
2.51	1.33[illegible]	.7080	.4880	-.2340	1.0136	.6986	-.6963
2.52	1.34[illegible]5	.7127	.4857	-.231[illegible]	1.0230	.6969	-.694[illegible]
2.53	1.3570	.7177	.4834	-.2296	1.0324	.6952	-.6833
2.54	1.3679	.7226	.4811	-.2275	1.0419	.6936	-.6818
2.55	1.3777	.7274	.4788	-.2254	1.0512	.6920	-.6803
2.56	1.3879	.7322	.4766	-.2233	1.0606	.6903	-.6789
2.57	1.3981	.7369	.4744	-.2213	1.0700	.6887	-.6773
2.58	1.40[illegible]4	.7416	.4722	-.2192	1.0793	.6871	-.6758
2.59	1.4190	.7464	.4700	-.2173	1.0887	.6855	-.6743
2.60	1.4296	.7510	.4678	-.2153	1.0981	.6840	-.6729
2.61	1.4404	.7557	.4657	-.2134	1.1074	.6824	-.6714
2.62	1.4514	.7604	.4635	-.2114	1.1168	.6803	-.6700
2.63	1.4625	.7650	.4614	-.2096	1.1262	.6793	-.6685
2.64	1.4738	.7696	.4593	-.2077	1.1355	.6778	-.6671
2.65	1.4852	.7742	.4573	-.2058	1.1448	.6762	-.6657
2.66	1.4968	.7787	.4552	-.2040	1.1542	.6747	-.6643
2.67	1.5085	.7833	.4532	-.2022	1.1635	.6732	-.6629
2.68	1.5204	.7879	.4512	-.2005	1.1728	.6717	-.6615
2.69	1.5325	.7923	.4492	-.1987	1.1822	.6702	-.6601
2.70	1.5447	.7968	.4472	-.1970	1.1915	.6687	-.6587
2.71	1.5571	.8012	.4453	-.1953	1.2008	.6673	-.6574
2.72	1.5696	.8057	.4433	-.1936	1.2101	.6658	-.6560
2.73	1.5824	.8101	.4414	-.1920	1.2194	.6644	-.6547
2.74	1.5953	.8145	.4395	-.1903	1.2287	.6629	-.6533
2.75	1.6084	.8189	.4376	-.1887	1.2380	.6615	-.6520
2.76	1.6216	.8233	.4357	-.1871	1.2472	.6601	-.6507
2.77	1.6351	.8276	.4339	-.1855	1.2565	.6587	-.6493
2.78	1.6487	.8319	.4320	-.1840	1.2658	.6573	-.6480
2.79	1.6625	.8363	.4301	-.1824	1.2751	.6559	-.6467
2.80	1.6765	.8405	.4283	-.1809	1.2843	.6545	-.6454
2.81	1.6907	.8448	.4265	-.1794	1.2936	.6531	-.6441
2.82	1.7051	.8491	.4247	-.1779	1.3028	.6517	-.6428
2.83	1.7196	.8533	.4230	-.1765	1.3121	.6504	-.6416
2.84	1.7344	.8575	.4212	-.1750	1.3213	.6490	-.6403
2.85	1.7494	.8617	.4195	-.1736	1.3305	.6477	-.6390
2.86	1.7646	.8659	.4177	-.1722	1.3398	.6463	-.6378
2.87	1.7799	.8701	.4160	-.1708	1.3490	.6450	-.6365
2.88	1.7955	.8742	.4143	-.1694	1.3582	.6437	-.6353
2.89	1.8113	.8784	.4126	-.1681	1.3674	.6424	-.6341
2.90	1.8274	.8825	.4110	-.1667	1.3766	.6411	-.6328
2.91	1.8436	.8866	.4093	-.1654	1.3858	.6398	-.6316
2.92	1.8600	.8907	.4077	-.1641	1.3950	.6385	-.6304
2.93	1.8767	.8943	.4060	-.1628	1.4042	.6372	-.6292
2.94	1.8936	.8988	.4044	-.1615	1.4134	.6359	-.6280
2.95	1.9108	.9028	.4028	-.1602	1.4226	.6347	-.6268
2.96	1.9281	.9069	.4012	-.1590	1.4317	.6334	-.6256
2.97	1.9457	.9109	.3996	-.1577	1.4409	.6321	-.6244
2.98	1.9636	.9149	.3980	-.1565	1.4501	.6309	-.6233
2.99	1.9[illegible]17	.9188	.3965	-.1553	1.4592	.6297	-.6221

X	GAMMA 1	PS	PS1	PS2	PS/VPS1	VPS1	PS2/PS1**3/2
3.00	2.0000	.9228	.3948	-.1541	1.4694	.6284	-.6209
3.01	2.0186	.9267	.3934	-.1529	1.4775	.6272	-.6198
3.02	2.0374	.9307	.3919	-.1519	1.4867	.6260	-.6186
3.03	2.0565	.9345	.3904	-.1505	1.4958	.6248	-.6175
3.04	2.0759	.9385	.3889	-.1495	1.5049	.6236	-.6164
3.05	2.0955	.9423	.3874	-.1483	1.5141	.6224	-.6152
3.06	2.1153	.9462	.3859	-.1472	1.5232	.6212	-.6141
3.07	2.1355	.9501	.3844	-.1461	1.5323	.6200	-.6130
3.08	2.1559	.9539	.3830	-.1450	1.5414	.6188	-.6119
3.09	2.1766	.9577	.3815	-.1439	1.5505	.6177	-.6108
3.10	2.1976	.9615	.3801	-.1429	1.5596	.6165	-.6097
3.11	2.2187	.9653	.3787	-.1418	1.5687	.6154	-.6086
3.12	2.2405	.9691	.3773	-.1408	1.5778	.6142	-.6075
3.13	2.2623	.9729	.3759	-.1397	1.5869	.6131	-.6064
3.14	2.2845	.9766	.3745	-.1387	1.5959	.6119	-.6053
3.15	2.3069	.9804	.3731	-.1377	1.6050	.6108	-.6043
3.16	2.3297	.9841	.3717	-.1367	1.6141	.6097	-.6032
3.17	2.3528	.9878	.3703	-.1357	1.6231	.6086	-.6021
3.18	2.3762	.9915	.3690	-.1347	1.6322	.6075	-.6011
3.19	2.3999	.9952	.3677	-.1338	1.6413	.6063	-.6000
3.20	2.4240	.9988	.3663	-.1328	1.6503	.6052	-.5990
3.21	2.4483	1.0025	.3650	-.1319	1.6593	.6042	-.5979
3.22	2.4730	1.0061	.3637	-.1309	1.6684	.6031	-.5969
3.23	2.4981	1.0098	.3624	-.1300	1.6774	.6020	-.5959
3.24	2.5235	1.0134	.3611	-.1291	1.6864	.6009	-.5948
3.25	2.5492	1.0170	.3598	-.1282	1.6955	.5998	-.5938
3.26	2.5754	1.0206	.3585	-.1273	1.7045	.5988	-.5928
3.27	2.6019	1.0242	.3573	-.1264	1.7135	.5977	-.5918
3.28	2.6286	1.0277	.3560	-.1255	1.7225	.5967	-.5908
3.29	2.6559	1.0313	.3547	-.1246	1.7315	.5956	-.5898
3.30	2.6834	1.0348	.3535	-.1238	1.7405	.5946	-.5888
3.31	2.7114	1.0384	.3523	-.1229	1.7495	.5935	-.5878
3.32	2.7397	1.0419	.3510	-.1221	1.7585	.5925	-.5868
3.33	2.7685	1.0454	.3498	-.1212	1.7674	.5915	-.5858
3.34	2.7976	1.0489	.3486	-.1204	1.7764	.5904	-.5849
3.35	2.8272	1.0523	.3474	-.1196	1.7854	.5894	-.5839
3.36	2.8571	1.0558	.3462	-.1188	1.7943	.5884	-.5829
3.37	2.8875	1.0593	.3450	-.1180	1.8033	.5874	-.5820
3.38	2.9183	1.0627	.3438	-.1172	1.8123	.5864	-.5810
3.39	2.9495	1.0661	.3427	-.1164	1.8212	.5854	-.5800
3.40	2.9812	1.0696	.3415	-.1156	1.8301	.5844	-.5791
3.41	3.0133	1.0730	.3404	-.1148	1.8391	.5834	-.5782
3.42	3.0459	1.0764	.3392	-.1141	1.8480	.5824	-.5772
3.43	3.0789	1.0798	.3381	-.1133	1.8569	.5815	-.5763
3.44	3.1124	1.0831	.3370	-.1125	1.8659	.5805	-.5754
3.45	3.1463	1.0865	.3359	-.1118	1.8748	.5795	-.5744
3.46	3.1807	1.0899	.3347	-.1111	1.8837	.5786	-.5735
3.47	3.2156	1.0932	.3336	-.1103	1.8926	.5776	-.5726
3.48	3.2510	1.0965	.3325	-.1096	1.9015	.5767	-.5717
3.49	3.2869	1.0998	.3314	-.1089	1.9104	.5757	-.5708

X	GAMMA ,	PS	PS1	PS2	PS/VPS1	VPS1	PS2/PS1**3/2
3.50	3.3233	1.1032	.3304	-.1082	1.9193	.5748	-.5699
3.51	3.3603	1.1065	.3293	-.1075	1.9282	.5738	-.5690
3.52	3.3977	1.1097	.3282	-.1068	1.9371	.5729	-.5681
3.53	3.4357	1.1130	.3271	-.1061	1.9460	.5720	-.5672
3.54	3.4742	1.1163	.3261	-.1054	1.9548	.5710	-.5663
3.55	3.5132	1.1195	.3250	-.1048	1.9637	.5701	-.5654
3.56	3.5529	1.1228	.3240	-.1041	1.9726	.5692	-.5645
3.57	3.5930	1.1260	.3230	-.1034	1.9814	.5683	-.5636
3.58	3.6339	1.1292	.3219	-.1028	1.9903	.5674	-.5628
3.59	3.6751	1.1325	.3209	-.1021	1.9991	.5665	-.5619
3.60	3.7170	1.1357	.3199	-.1015	2.0080	.5656	-.5610
3.61	3.7595	1.1389	.3189	-.1009	2.0168	.5647	-.5602
3.62	3.8026	1.1420	.3179	-.1002	2.0256	.5638	-.5593
3.63	3.8464	1.1452	.3161	-.0996	2.0345	.5629	-.5585
3.64	3.8907	1.1484	.3159	-.0990	2.0433	.5620	-.5576
3.65	3.9357	1.1515	.3149	-.0984	2.0521	.5611	-.5568
3.66	3.9814	1.1547	.3139	-.0978	2.0609	.5603	-.5559
3.67	4.0277	1.1578	.3129	-.0972	2.0697	.5594	-.5551
3.68	4.0747	1.1609	.3120	-.0966	2.0785	.5585	-.5543
3.69	4.1223	1.1640	.3110	-.0960	2.0873	.5577	-.5534
3.70	4.1706	1.1672	.3100	-.0954	2.0961	.5568	-.5526
3.71	4.2197	1.1702	.3091	-.0948	2.1049	.5560	-.5518
3.72	4.2694	1.1733	.3081	-.0942	2.1137	.5551	-.5509
3.73	4.3199	1.1764	.3072	-.0937	2.1225	.5543	-.5501
3.74	4.3710	1.1795	.3063	-.0931	2.1313	.5534	-.5493
3.75	4.4230	1.1825	.3053	-.0925	2.1400	.5526	-.5485
3.76	4.4757	1.1856	.3044	-.0920	2.1488	.5517	-.5477
3.77	4.5291	1.1886	.3035	-.0914	2.1576	.5509	-.5469
3.78	4.5833	1.1917	.3026	-.0909	2.1663	.5501	-.5461
3.79	4.6383	1.1947	.3017	-.0904	2.1751	.5493	-.5453
3.80	4.6942	1.1977	.3008	-.0898	2.1838	.5484	-.5445
3.81	4.7508	1.2007	.2999	-.0893	2.1926	.5476	-.5437
3.82	4.8082	1.2037	.2990	-.0888	2.2013	.5468	-.5429
3.83	4.8665	1.2067	.2981	-.0882	2.2100	.5460	-.5422
3.84	4.9257	1.2096	.2972	-.0877	2.2188	.5452	-.5414
3.85	4.9857	1.2126	.2964	-.0872	2.2275	.5444	-.5406
3.86	5.0466	1.2156	.2955	-.0867	2.2362	.5436	-.5398
3.87	5.1084	1.2185	.2946	-.0862	2.2449	.5428	-.5391
3.88	5.1711	1.2215	.2938	-.0857	2.2536	.5420	-.5383
3.89	5.2347	1.2244	.2929	-.0852	2.2623	.5412	-.5375
3.90	5.2993	1.2273	.2921	-.0847	2.2710	.5404	-.5368
3.91	5.3648	1.2302	.2912	-.0842	2.2797	.5396	-.5360
3.92	5.4313	1.2332	.2904	-.0838	2.2884	.5389	-.5353
3.93	5.4989	1.2361	.2895	-.0833	2.2971	.5381	-.5345
3.94	5.5673	1.2389	.2887	-.0828	2.3058	.5373	-.5338
3.95	5.6367	1.2418	.2879	-.0823	2.3145	.5365	-.5330
3.96	5.7073	1.2447	.2871	-.0817	2.3232	.5358	-.5323
3.97	5.7788	1.2476	.2862	-.0814	2.3318	.5350	-.5315
3.98	5.8515	1.2504	.2854	-.0809	2.3405	.5343	-.5308
3.99	5.9252	1.2533	.2846	-.0805	2.3491	.5335	-.5301

X	GAMMA ,	PS	PS1	PS2	PS/VPS1	VPS1	PS2/PS1**3/2
4.00	6.0000	1.2561	.2836	-.0800	2.3578	.5328	-.5293
4.01	6.0757	1.2590	.2830	-.0796	2.3664	.5320	-.5286
4.02	6.1530	1.2618	.2822	-.0792	2.3751	.5313	-.5279
4.03	6.2312	1.2646	.2814	-.0787	2.3837	.5305	-.5272
4.04	6.3106	1.2674	.2807	-.0783	2.3924	.5298	-.5265
4.05	6.3911	1.2702	.2799	-.0778	2.4010	.5290	-.5257
4.06	6.4721	1.2730	.2791	-.0774	2.4096	.5283	-.5250
4.07	6.5560	1.2758	.2783	-.0770	2.4182	.5276	-.5243
4.08	6.6402	1.2786	.2776	-.0766	2.4269	.5268	-.5236
4.09	6.7259	1.2813	.2768	-.0761	2.4355	.5261	-.5229
4.10	6.8126	1.2841	.2760	-.0757	2.4441	.5254	-.5222
4.11	6.9007	1.2869	.2753	-.0753	2.4527	.5247	-.5215
4.12	6.9902	1.2896	.2745	-.0749	2.4613	.5240	-.5208
4.13	7.0810	1.2924	.2738	-.0745	2.4699	.5232	-.5201
4.14	7.1732	1.2951	.2730	-.0741	2.4785	.5225	-.5194
4.15	7.2669	1.2978	.2723	-.0737	2.4871	.5218	-.5187
4.16	7.3619	1.3005	.2716	-.0733	2.4956	.5211	-.5181
4.17	7.4583	1.3032	.2708	-.0729	2.5042	.5204	-.5174
4.18	7.5563	1.3060	.2701	-.0725	2.5128	.5197	-.5167
4.19	7.6557	1.3086	.2694	-.0721	2.5214	.5190	-.5160
4.20	7.7567	1.3113	.2687	-.0718	2.5299	.5183	-.5153
4.21	7.8591	1.3140	.2679	-.0714	2.5385	.5176	-.5147
4.22	7.9632	1.3167	.2672	-.0710	2.5470	.5170	-.5140
4.23	8.0689	1.3194	.2665	-.0706	2.5556	.5163	-.5133
4.24	8.1761	1.3220	.2658	-.0703	2.5641	.5156	-.5127
4.25	8.2850	1.3247	.2651	-.0699	2.5727	.5149	-.5120
4.26	8.3956	1.3273	.2644	-.0695	2.5812	.5142	-.5114
4.27	8.5079	1.3300	.2637	-.0692	2.5898	.5135	-.5107
4.28	8.6220	1.3326	.2630	-.0688	2.5983	.5129	-.5100
4.29	8.7377	1.3352	.2624	-.0685	2.6068	.5122	-.5094
4.30	8.8553	1.3379	.2617	-.0681	2.6153	.5115	-.5087
4.31	8.9747	1.3405	.2610	-.0677	2.6239	.5109	-.5081
4.32	9.0959	1.3431	.2603	-.0674	2.6324	.5102	-.5074
4.33	9.2190	1.3457	.2596	-.0671	2.6409	.5096	-.5068
4.34	9.3440	1.3483	.2590	-.0667	2.6494	.5089	-.5062
4.35	9.4710	1.3509	.2583	-.0664	2.6579	.5082	-.5055
4.36	9.5998	1.3534	.2577	-.0660	2.6664	.5076	-.5049
4.37	9.7308	1.3560	.2570	-.0657	2.6749	.5069	-.5043
4.38	9.8639	1.3586	.2563	-.0654	2.6833	.5063	-.5036
4.39	9.9989	1.3611	.2557	-.0650	2.6918	.5057	-.5030
4.40	10.1360	1.3637	.2550	-.0647	2.7003	.5050	-.5024
4.41	10.2754	1.3662	.2544	-.0644	2.7088	.5044	-.5018
4.42	10.4169	1.3688	.2537	-.0641	2.7172	.5037	-.5011
4.43	10.5605	1.3713	.2531	-.0637	2.7257	.5031	-.5005
4.44	10.7065	1.3738	.2525	-.0634	2.7342	.5025	-.4999
4.45	10.8547	1.3764	.2518	-.0631	2.7426	.5018	-.4993
4.46	11.0053	1.3789	.2512	-.0628	2.7511	.5012	-.4987
4.47	11.1582	1.3814	.2506	-.0625	2.7595	.5006	-.4981
4.48	11.3136	1.3839	.2500	-.0622	2.7680	.5000	-.4975
4.49	11.4714	1.3864	.2493	-.0619	2.7764	.4993	-.4968

X	GAMMA	PS	PS1	PS2	PS/VPS1	VPS1	PS2/PS1**3/2
4.50	11.6317	1.3839	.2487	-.0616	2.7849	.4987	-.4962
4.51	11.7[illegible]45	1.3914	.2481	-.0613	2.7933	.4981	-.4956
4.52	11.95[illegible]	1.3938	.2475	-.0610	2.8017	.4975	-.4950
4.53	12.1279	1.3963	.2469	-.0607	2.8101	.4969	-.4944
4.54	12.2[illegible]86	1.3988	.2463	-.0604	2.8185	.4963	-.4939
4.55	12.4720	1.4012	.2457	-.0601	2.8270	.4957	-.4933
4.56	12.6481	1.4037	.2451	-.0598	2.8354	.4951	-.4927
4.57	12.8271	1.4061	.2445	-.0595	2.8438	.4945	-.4921
4.58	13.0089	1.4086	.2439	-.0592	2.8522	.4939	-.4915
4.59	13.1936	1.4110	.2433	-.0589	2.8606	.4933	-.4909
4.60	13.3812	1.4134	.2427	-.0586	2.8690	.4927	-.4903
4.61	13.5719	1.4159	.2421	-.0584	2.8774	.4921	-.4897
4.62	13.7656	1.4183	.2416	-.0581	2.8857	.4915	-.4892
4.63	13.9623	1.4207	.2410	-.0578	2.8941	.4909	-.4886
4.64	14.1623	1.4231	.2404	-.0575	2.9025	.4903	-.4880
4.65	14.3654	1.4255	.2398	-.0572	2.9109	.4897	-.4874
4.66	14.5719	1.4279	.2392	-.0570	2.9193	.4891	-.4869
4.67	14.7816	1.4303	.2387	-.0567	2.9276	.4886	-.4863
4.68	14.9947	1.4327	.2381	-.0564	2.9360	.4880	-.4857
4.69	15.2113	1.4351	.2376	-.0562	2.9443	.4874	-.4852
4.70	15.4313	1.4374	.2370	-.0559	2.9527	.4868	-.4846
4.71	15.6549	1.4398	.2364	-.0556	2.9610	.4862	-.4841
4.72	15.8821	1.4422	.2359	-.0554	2.9694	.4857	-.4835
4.73	16.1130	1.4445	.2353	-.0551	2.9777	.4851	-.4829
4.74	16.3477	1.4469	.2348	-.0549	2.9861	.4845	-.4824
4.75	16.5861	1.4492	.2342	-.0546	2.9944	.4840	-.4818
4.76	16.8284	1.4515	.2337	-.0544	3.0027	.4834	-.4813
4.77	17.0747	1.4539	.2331	-.0541	3.0110	.4828	-.4807
4.78	17.3249	1.4562	.2326	-.0539	3.0194	.4823	-.4802
4.79	17.5793	1.4585	.2321	-.0536	3.0277	.4817	-.4796
4.80	17.8378	1.4608	.2315	-.0534	3.0360	.4812	-.4791
4.81	18.1005	1.4632	.2310	-.0531	3.0443	.4806	-.4786
4.82	18.3675	1.4655	.2305	-.0529	3.0526	.4801	-.4780
4.83	18.6389	1.4678	.2299	-.0526	3.0609	.4795	-.4775
4.84	18.9146	1.4701	.2294	-.0524	3.0692	.4790	-.4769
4.85	19.1950	1.4724	.2289	-.0522	3.0775	.4784	-.4764
4.86	19.4799	1.4746	.2284	-.0519	3.0858	.4779	-.4759
4.87	19.7695	1.4769	.2279	-.0517	3.0941	.4773	-.4753
4.88	20.0639	1.4792	.2273	-.0515	3.1024	.4768	-.4748
4.89	20.3631	1.4815	.2268	-.0512	3.1106	.4763	-.4743
4.90	20.6673	1.4837	.2263	-.0510	3.1189	.4757	-.4738
4.91	20.9764	1.4[illegible]60	.2258	-.0508	3.1272	.4752	-.4732
4.92	21.2907	1.4883	.2253	-.0506	3.1355	.4747	-.4727
4.93	21.6102	1.4905	.2248	-.0503	3.1437	.4741	-.4722
4.94	21.9349	1.4927	.2243	-.0501	3.1520	.4736	-.4717
4.95	22.2651	1.4950	.2239	-.0499	3.1602	.4731	-.4711
4.96	22.6007	1.4972	.2233	-.0497	3.1685	.4725	-.4706
4.97	22.941[illegible]	1.4995	.2228	-.0494	3.1767	.4720	-.4701
4.98	23.2887	1.5017	.2223	-.0492	3.1850	.4715	-.4696
4.99	23.6414	1.5039	.2218	-.0490	3.1932	.4710	-.4691

X	GAMMA ,	PS	PS1	PS2	PS/VPS1	VPS1	PS2/PS1**3/2
5.00	23.9958	1.5061	.2215	-.0489	3.2014	.4704	-.4686
5.01	24.3843	1.5083	.2209	-.0486	3.2097	.4699	-.4681
5.02	24.7344	1.5105	.2204	-.0484	3.2179	.4694	-.4676
5.03	25.1116	1.5127	.2199	-.0482	3.2261	.4689	-.4671
5.04	25.4947	1.5149	.2194	-.0479	3.2343	.4684	-.4666
5.05	25.8841	1.5171	.2189	-.0477	3.2426	.4679	-.4661
5.06	26.2801	1.5193	.2184	-.0475	3.2508	.4674	-.4656
5.07	26.6827	1.5215	.2179	-.0473	3.2590	.4669	-.4651
5.08	27.0921	1.5237	.2175	-.0471	3.2672	.4664	-.4646
5.09	27.5083	1.5258	.2170	-.0469	3.2754	.4659	-.4641
5.10	27.9316	1.5280	.2165	-.0467	3.2836	.4653	-.4636
5.11	28.3619	1.5302	.2161	-.0465	3.2918	.4648	-.4631
5.12	28.7996	1.5323	.2156	-.0463	3.3000	.4643	-.4626
5.13	29.2446	1.5345	.2152	-.0461	3.3081	.4638	-.4621
5.14	29.6971	1.5366	.2147	-.0459	3.3163	.4634	-.4616
5.15	30.1573	1.5388	.2142	-.0457	3.3245	.4629	-.4611
5.16	30.6253	1.5409	.2138	-.0455	3.3327	.4624	-.4607
5.17	31.1012	1.5431	.2133	-.0453	3.3409	.4619	-.4602
5.18	31.5852	1.5452	.2129	-.0451	3.3490	.4614	-.4597
5.19	32.0773	1.5473	.2124	-.0450	3.3572	.4609	-.4592
5.20	32.5779	1.5494	.2120	-.0448	3.3653	.4604	-.4587
5.21	33.0869	1.5516	.2115	-.0446	3.3735	.4599	-.4583
5.22	33.6046	1.5537	.2111	-.0444	3.3817	.4594	-.4578
5.23	34.1312	1.5558	.2106	-.0442	3.3898	.4590	-.4573
5.24	34.6667	1.5579	.2102	-.0440	3.3979	.4585	-.4568
5.25	35.2114	1.5600	.2098	-.0438	3.4061	.4580	-.4564
5.26	35.7653	1.5621	.2093	-.0437	3.4142	.4575	-.4559
5.27	36.3288	1.5642	.2089	-.0435	3.4224	.4570	-.4554
5.28	36.9019	1.5662	.2085	-.0433	3.4305	.4566	-.4550
5.29	37.4848	1.5683	.2080	-.0431	3.4386	.4561	-.4545
5.30	38.0777	1.5704	.2076	-.0429	3.4467	.4556	-.4540
5.31	38.6808	1.5725	.2072	-.0428	3.4549	.4552	-.4536
5.32	39.2943	1.5746	.2067	-.0426	3.4630	.4547	-.4531
5.33	39.9183	1.5766	.2063	-.0424	3.4711	.4542	-.4526
5.34	40.5531	1.5787	.2059	-.0422	3.4792	.4537	-.4522
5.35	41.1989	1.5807	.2055	-.0421	3.4873	.4533	-.4517
5.36	41.8556	1.5828	.2050	-.0419	3.4954	.4528	-.4513
5.37	42.5238	1.5848	.2046	-.0417	3.5035	.4524	-.4508
5.38	43.2035	1.5869	.2042	-.0416	3.5116	.4519	-.4504
5.39	43.8951	1.5889	.2038	-.0414	3.5197	.4514	-.4499
5.40	44.5985	1.5910	.2034	-.0412	3.5278	.4510	-.4495
5.41	45.3142	1.5930	.2030	-.0411	3.5359	.4505	-.4490
5.42	46.0423	1.5950	.2026	-.0409	3.5439	.4501	-.4486
5.43	46.7830	1.5970	.2022	-.0407	3.5520	.4496	-.4481
5.44	47.5367	1.5991	.2017	-.0406	3.5601	.4492	-.4477
5.45	48.3034	1.6011	.2013	-.0404	3.5682	.4487	-.4472
5.46	49.0835	1.6031	.2009	-.0402	3.5762	.4483	-.4468
5.47	49.8772	1.6051	.2005	-.0401	3.5843	.4478	-.4464
5.48	50.6847	1.6071	.2001	-.0399	3.5923	.4474	-.4459
5.49	51.5064	1.6091	.1997	-.0398	3.6004	.4469	-.4455

X	GAMMA ,	PS	PS1	PS2	PS/VPS1	VPS1	PS2/PS1**3/2
5.50	52.3424	1.6111	.1995	-.0396	3.6084	.4465	-.4450
5.51	53.1930	1.6131	.1991	-.0395	3.6165	.4460	-.4446
5.52	54.0566	1.6151	.1986	-.0393	3.6245	.4456	-.4442
5.53	54.9373	1.6171	.1982	-.0391	3.6326	.4452	-.4437
5.54	55.8359	1.6190	.1978	-.0390	3.6406	.4447	-.4433
5.55	56.7474	1.6210	.1974	-.0388	3.6487	.4443	-.4429
5.56	57.6713	1.6230	.1970	-.0387	3.6567	.4438	-.4424
5.57	58.6156	1.6250	.1966	-.0385	3.6647	.4434	-.4420
5.58	59.5805	1.6269	.1962	-.0384	3.6727	.4430	-.4416
5.59	60.5583	1.6289	.1959	-.0382	3.6808	.4425	-.4412
5.60	61.5534	1.6308	.1955	-.0381	3.6888	.4421	-.4407
5.61	62.5661	1.6328	.1951	-.0379	3.6968	.4417	-.4403
5.62	63.5767	1.6347	.1947	-.0378	3.7048	.4412	-.4399
5.63	64.6455	1.6367	.1943	-.0376	3.7128	.4408	-.4395
5.64	65.7129	1.6386	.1939	-.0375	3.7208	.4404	-.4390
5.65	66.7992	1.6406	.1936	-.0374	3.7288	.4400	-.4386
5.66	67.9047	1.6425	.1932	-.0372	3.7368	.4395	-.4382
5.67	69.0300	1.6444	.1928	-.0371	3.7449	.4391	-.4378
5.68	70.1751	1.6463	.1925	-.0369	3.7528	.4387	-.4374
5.69	71.3407	1.6483	.1921	-.0368	3.7608	.4383	-.4370
5.70	72.5270	1.6502	.1917	-.0366	3.7687	.4379	-.4366
5.71	73.7345	1.6521	.1914	-.0365	3.7767	.4374	-.4361
5.72	74.9635	1.6540	.1910	-.0364	3.7847	.4370	-.4357
5.73	76.2145	1.6559	.1906	-.0362	3.7927	.4366	-.4353
5.74	77.4878	1.6578	.1903	-.0361	3.8006	.4362	-.4349
5.75	78.7839	1.6597	.1899	-.0360	3.8086	.4358	-.4345
5.76	80.1031	1.6616	.1895	-.0358	3.8166	.4354	-.4341
5.77	81.4460	1.6635	.1892	-.0357	3.8245	.4350	-.4337
5.78	82.8130	1.6654	.1888	-.0356	3.8325	.4346	-.4333
5.79	84.2045	1.6673	.1885	-.0354	3.8404	.4341	-.4329
5.80	85.6210	1.6692	.1881	-.0353	3.8484	.4337	-.4325
5.81	87.0630	1.6711	.1878	-.0352	3.8563	.4333	-.4321
5.82	88.5307	1.6729	.1874	-.0350	3.8643	.4329	-.4317
5.83	90.0253	1.6748	.1871	-.0349	3.8722	.4325	-.4313
5.84	91.5466	1.6767	.1867	-.0348	3.8801	.4321	-.4309
5.85	93.0954	1.6785	.1864	-.0346	3.8881	.4317	-.4305
5.86	94.6721	1.6804	.1860	-.0345	3.8960	.4313	-.4301
5.87	96.2773	1.6823	.1857	-.0344	3.9039	.4309	-.4297
5.88	97.9116	1.6841	.1853	-.0343	3.9119	.4305	-.4293
5.89	99.5753	1.6860	.1850	-.0341	3.9198	.4301	-.4289
5.90	101.2693	1.6878	.1847	-.0340	3.9277	.4297	-.4285
5.91	102.9941	1.6897	.1843	-.0339	3.9356	.4293	-.4281
5.92	104.7500	1.6915	.1840	-.0338	3.9435	.4289	-.4277
5.93	106.5377	1.6933	.1836	-.0336	3.9514	.4285	-.4274
5.94	108.3583	1.6952	.1833	-.0335	3.9593	.4281	-.4270
5.95	110.2119	1.6970	.1830	-.0334	3.9672	.4278	-.4266
5.96	112.0992	1.6988	.1826	-.0333	3.9751	.4274	-.4262
5.97	114.0203	1.7007	.1823	-.0331	3.9830	.4270	-.4258
5.98	115.9776	1.7025	.1820	-.0330	3.9909	.4266	-.4254
5.99	117.9701	1.7043	.1817	-.0329	3.9988	.4262	-.4251

X	GAMMA	PS	PS1	PS2	PS/VPS1	VPS1	PS2/PS1**3/2
6.00	11[illegible].5[illegible]0	1.7061	.1813	-.0329	4.0067	.4258	-.4247
6.01	122.0659	1.7079	.1810	-.0327	4.0145	.4254	-.4243
6.02	124.16[illegible]	1.7097	.1807	-.0326	4.0224	.4251	-.4239
6.03	126.3111	1.7115	.1803	-.0324	4.0303	.4247	-.4235
6.04	12[illegible].4[illegible]27	1.7133	.1800	-.0323	4.0382	.4243	-.4232
6.05	130.7144	1.7151	.1797	-.0322	4.0460	.4239	-.4228
6.06	132.975[illegible]	1.7169	.1794	-.0321	4.0539	.4235	-.4224
6.07	135.2[illegible]0[illegible]	1.7187	.1791	-.0320	4.0617	.4232	-.4220
6.08	137.6274	1.7205	.1787	-.0319	4.0696	.4228	-.4217
6.09	140.0170	1.7223	.1784	-.0318	4.0775	.4224	-.4213
6.10	142.4507	1.7241	.1781	-.0316	4.0853	.4220	-.4209
6.11	144.9[illegible]	1.7259	.1778	-.0315	4.0932	.4216	-.4206
6.12	147.4536	1.7276	.1775	-.0314	4.1010	.4213	-.4202
6.13	150.0244	1.7294	.1772	-.0313	4.1088	.4209	-.4199
6.14	152.6429	1.7312	.1768	-.0312	4.1167	.4205	-.4195
6.15	155.3098	1.7330	.1765	-.0311	4.1245	.4202	-.4191
6.16	158.0261	1.7347	.1762	-.0310	4.1323	.4198	-.4187
6.17	160.7929	1.7365	.1759	-.0309	4.1402	.4194	-.4184
6.18	163.6107	1.7382	.1756	-.0308	4.1480	.4191	-.4180
6.19	166.4809	1.7400	.1753	-.0307	4.1558	.4187	-.4176
6.20	169.4045	1.7417	.1750	-.0305	4.1636	.4183	-.4173
6.21	172.3824	1.7435	.1747	-.0304	4.1715	.4180	-.4169
6.22	175.4159	1.7452	.1744	-.0303	4.1793	.4176	-.4166
6.23	178.5057	1.7470	.1741	-.0302	4.1871	.4172	-.4162
6.24	181.6531	1.7487	.1738	-.0301	4.1949	.4169	-.4159
6.25	184.8592	1.7505	.1735	-.0300	4.2027	.4165	-.4155
6.26	188.1252	1.7522	.1732	-.0299	4.2105	.4161	-.4151
6.27	191.4522	1.7539	.1729	-.0298	4.2183	.4158	-.4148
6.28	194.8415	1.7556	.1726	-.0297	4.2261	.4154	-.4144
6.29	198.2941	1.7574	.1723	-.0296	4.2339	.4151	-.4141
6.30	201.8113	1.7591	.1720	-.0295	4.2416	.4147	-.4137
6.31	205.3945	1.7608	.1717	-.0294	4.2494	.4144	-.4134
6.32	209.0450	1.7625	.1714	-.0293	4.2572	.4140	-.4130
6.33	212.7640	1.7642	.1711	-.0292	4.2650	.4137	-.4127
6.34	216.5528	1.7659	.1708	-.0291	4.2728	.4133	-.4123
6.35	220.4129	1.7677	.1705	-.0290	4.2805	.4130	-.4120
6.36	224.3455	1.7694	.1702	-.0289	4.2883	.4126	-.4116
6.37	228.3522	1.7711	.1699	-.0288	4.2961	.4122	-.4113
6.38	232.4344	1.7728	.1697	-.0287	4.3038	.4119	-.4109
6.39	236.5[illegible]37	1.7745	.1694	-.0286	4.3116	.4116	-.4106
6.40	240.8315	1.7761	.1691	-.0285	4.3194	.4112	-.4102
6.41	24[illegible].14[illegible]3	1.7778	.1688	-.0284	4.3271	.4109	-.4099
6.42	24[illegible].54[illegible]6	1.7795	.1685	-.0283	4.3349	.4105	-.4096
6.43	254.0313	1.7812	.1682	-.0282	4.3426	.4102	-.4092
6.44	25[illegible].598[illegible]	1.7829	.1680	-.0281	4.3504	.4098	-.4089
6.45	263.2[illegible]	1.7846	.1677	-.0281	4.3581	.4095	-.4085
6.46	267.9[illegible]	1.7862	.1674	-.0280	4.3659	.4091	-.4082
6.47	272.8[illegible]76	1.7879	.1671	-.0279	4.3736	.4088	-.4079
6.48	277.7515	1.7896	.1668	-.0278	4.3813	.4085	-.4075
6.49	2[illegible]2.76[illegible]2	1.7912	.1666	-.0277	4.3890	.4081	-.4072

M	[illegible]	[illegible]	[illegible]1	[illegible]	PS2/VPS1	VPS1	PS2/PS1**3/2
6.50	[illegible]	1.7[illegible]	.1663	-.0276	4.3969	.4073	-.4063
6.51	[illegible]	1.7[illegible]46	.1660	-.0275	4.4045	.4074	-.4065
6.52	[illegible].4029	1.7[illegible]62	.1657	-.0274	4.4122	.4071	-.4062
6.53	303.9138	1.7679	.1655	-.0273	4.4199	.4068	-.4059
6.54	308.5976	1.7[illegible]95	.1652	-.0272	4.4276	.4064	-.4055
6.55	314.9471	1.8012	.1649	-.0271	4.4353	.4061	-.4052
6.56	320.6731	1.80[illegible]	.1646	-.0270	4.4431	.4058	-.4049
6.57	32[illegible].5101	1.8045	.1644	-.0270	4.4508	.4054	-.4045
6.58	332.4[illegible]83	1.8061	.1641	-.0269	4.458[illegible]	.4051	-.4042
6.59	33[illegible].5203	1.8078	.1639	-.0268	4.4662	.4048	-.4039
6.60	344.6[illegible]84	1.8094	.1636	-.0267	4.4739	.4044	-.4036
6.61	350.9947	1.8110	.1633	-.0266	4.4816	.4041	-.4032
6.62	357.4125	1.8127	.1630	-.0265	4.4892	.4038	-.4029
6.63	363.9532	1.8143	.1628	-.0264	4.4969	.4035	-.4026
6.64	370.6196	1.8159	.1625	-.0264	4.5046	.4031	-.4023
6.65	377.4143	1.8175	.1622	-.0263	4.5123	.4028	-.4019
6.66	384.33[illegible]	1.8192	.1620	-.0262	4.5200	.4025	-.4016
6.67	391.3[illegible]89	1.8208	.1617	-.0261	4.5277	.4021	-.4013
6.68	398.5[illegible]39	1.8224	.1615	-.0260	4.5353	.4018	-.4010
6.69	405.9278	1.8240	.1612	-.0259	4.5430	.4015	-.4006
6.70	413.4032	1.8256	.1609	-.0258	4.5507	.4012	-.4003
6.71	421.0230	1.8272	.1607	-.0258	4.5583	.4009	-.4000
6.72	428.7903	1.8288	.1604	-.0257	4.5660	.4005	-.3997
6.73	436.7079	1.8304	.1602	-.0256	4.5736	.4002	-.3994
6.74	444.7757	1.8320	.1599	-.0255	4.5813	.3999	-.3991
6.75	453.0061	1.8336	.1597	-.0254	4.5890	.3996	-.3987
6.76	461.3929	1.8352	.1594	-.0254	4.5966	.3993	-.3984
6.77	469.9424	1.8368	.1592	-.0253	4.6043	.3989	-.3981
6.78	478.6580	1.8384	.1589	-.0252	4.6119	.3986	-.3978
6.79	487.5429	1.8400	.1587	-.0251	4.6195	.3983	-.3975
6.80	496.6007	1.8416	.1584	-.0250	4.6272	.3980	-.3972
6.81	505.8343	1.8432	.1581	-.0250	4.6348	.3977	-.3969
6.82	515.2486	1.8448	.1579	-.0249	4.6424	.3974	-.3966
6.83	524.8463	1.8463	.1577	-.0248	4.6501	.3971	-.3962
6.84	534.6309	1.8479	.1574	-.0247	4.6577	.3967	-.3959
6.85	544.6066	1.8495	.1572	-.0246	4.6653	.3964	-.3956
6.86	554.7767	1.8511	.1569	-.0246	4.6730	.3961	-.3953
6.87	565.1461	1.8526	.1567	-.0245	4.6806	.3958	-.3950
6.88	575.7184	1.8542	.1564	-.0244	4.6882	.3955	-.3947
6.89	586.4974	1.8557	.1562	-.0243	4.6958	.3952	-.3944
6.90	597.4876	1.8573	.1559	-.0243	4.7034	.3949	-.3941
6.91	608.6933	1.8588	.1557	-.0242	4.7110	.3946	-.3938
6.92	620.1196	1.8604	.1555	-.0241	4.7186	.3943	-.3935
6.93	631.76[illegible]3	1.8620	.1552	-.0240	4.7262	.3940	-.3932
6.94	643.6469	1.8635	.1550	-.0240	4.7338	.3937	-.3929
6.95	655.7591	1.8651	.1547	-.0239	4.7414	.3934	-.3926
6.96	668.1033	1.8666	.1545	-.0238	4.7490	.3931	-.3923
6.97	680.7076	1.8682	.1543	-.0237	4.7566	.3928	-.3920
6.98	693.5442	1.8697	.1540	-.0237	4.7642	.3925	-.3917
6.99	70[illegible].638[illegible]	1.8712	.1538	-.0236	4.7718	.3921	-.3914

[illegible]	GAMMA	PS	PS1	PS2	PS/VPS1	VPS1	PS2/PS1**3/2
7.00	720.0	1.8728	.1535	-.0235	4.7794	.3918	-.3911
7.01	733.[illegible]	1.8743	.1533	-.0235	4.7869	.3915	-.3909
7.02	747.5	1.8759	.1531	-.0234	4.7945	.3912	-.3905
7.03	761.7	1.8774	.152[illegible]	-.0233	4.8021	.3910	-.3902
7.04	77[illegible].1	1.8789	.1526	-.0232	4.8097	.3907	-.3899
7.05	790.3	1.8804	.1524	-.0232	4.8172	.3904	-.3896
7.06	[illegible]05.[illegible]	1.8820	.1521	-.0231	4.8249	.3901	-.3893
7.07	821.2	1.8835	.1519	-.0230	4.8324	.3898	-.3890
7.08	836.[illegible]	1.8850	.1517	-.0230	4.8399	.3895	-.3887
7.09	852.7	1.8865	.1515	-.0229	4.8475	.3892	-.3884
7.10	867.0	1.8880	.1512	-.0228	4.8550	.3889	-.3881
7.11	885.5	1.8895	.1510	-.0228	4.8626	.3886	-.3879
7.12	902.4	1.8910	.1507	-.0227	4.8701	.3883	-.3876
7.13	919.7	1.8925	.1505	-.0226	4.8777	.3880	-.3873
7.14	937.2	1.8941	.1503	-.0226	4.8852	.3877	-.3870
7.15	955.2	1.8956	.1501	-.0225	4.8928	.3874	-.3867
7.16	973.4	1.8971	.1499	-.0224	4.9003	.3871	-.3864
7.17	992.1	1.8986	.1496	-.0224	4.9079	.3868	-.3861
7.18	1011.1	1.9000	.1494	-.0223	4.9154	.3866	-.3859
7.19	1030.5	1.9015	.1492	-.0222	4.9229	.3863	-.3856
7.20	1050.3	1.9030	.1490	-.0222	4.9304	.3860	-.3853
7.21	1070.5	1.9045	.1488	-.0221	4.9379	.3857	-.3850
7.22	1091.1	1.9060	.1485	-.0220	4.9455	.3854	-.3847
7.23	1112.1	1.9075	.1483	-.0220	4.9530	.3851	-.3844
7.24	1133.5	1.9090	.1481	-.0219	4.9605	.3848	-.3841
7.25	1155.4	1.9105	.1479	-.0218	4.9680	.3846	-.3839
7.26	1177.7	1.9119	.1477	-.0218	4.9755	.3843	-.3836
7.27	1200.4	1.9134	.1474	-.0217	4.9830	.3840	-.3833
7.28	1223.6	1.9149	.1472	-.0216	4.9905	.3837	-.3830
7.29	1247.3	1.9164	.1470	-.0216	4.9980	.3834	-.3827
7.30	1271.4	1.9178	.1468	-.0215	5.0055	.3831	-.3825
7.31	1296.1	1.9193	.1466	-.0214	5.0130	.3829	-.3822
7.32	1321.2	1.9208	.1464	-.0214	5.0205	.3826	-.3819
7.33	1346.[illegible]	1.9222	.1462	-.0213	5.0280	.3823	-.3816
7.34	1373.0	1.9237	.1459	-.0213	5.0355	.3820	-.3814
7.35	1399.6	1.9251	.1457	-.0212	5.0430	.3817	-.3811
7.36	1426.[illegible]	1.9266	.1455	-.0211	5.0505	.3815	-.3808
7.37	1454.6	1.9280	.1453	-.0211	5.0580	.3812	-.3805
7.38	1482.[illegible]	1.9295	.1451	-.0210	5.0654	.3809	-.3803
7.39	1511.5	1.9309	.1449	-.0210	5.0729	.3806	-.3800
7.40	1541.3	1.9324	.1447	-.0209	5.0804	.3804	-.3797
7.41	1571.4	1.9338	.1445	-.0208	5.0879	.3801	-.3794
7.42	1602.1	1.9353	.1443	-.0208	5.0953	.3798	-.3792
7.43	1633.4	1.9367	.1441	-.0207	5.1028	.3795	-.3789
7.44	1665.4	1.9382	.1439	-.0207	5.1103	.3793	-.3786
7.45	1698.0	1.9396	.1436	-.0206	5.1177	.3790	-.3784
7.46	1731.3	1.9410	.1434	-.0205	5.1252	.3787	-.3781
7.47	1765.2	1.9425	.1432	-.0205	5.1326	.3785	-.3778
7.48	179[illegible].[illegible]	1.9439	.1430	-.0204	5.1401	.3782	-.3775
7.49	1835.2	1.9453	.1428	-.0204	5.1475	.3779	-.3773

[illegible]	[illegible]	[illegible]	[illegible]31	[illegible]	[illegible]/VPS1	VPS1	PS2/PS1**3/[illegible]
7.50	1871.3	1.946[illegible]	.1425	-.0203	5.1550	.3776	-.3770
7.51	1909.0	1.9482	.1424	-.0202	5.1624	.3774	-.3767
7.52	1947.[illegible]	1.9496	.1422	-.0202	5.1698	.3771	-.3765
7.53	[illegible]	1.9510	.1420	-.0201	5.1773	.3769	-.3762
7.54	2025.9	1.9524	.141	-.0201	5.1847	.3766	-.3760
7.55	2062.[illegible]	1.9538	.1415	-.0200	5.1922	.3763	-.3757
7.56	2103.[illegible]	1.9553	.1414	-.0200	5.1996	.3760	-.3754
7.57	2145.2	1.9567	.1412	-.0199	5.2071	.3758	-.3752
7.58	2187.[illegible]	1.9581	.1410	-.0199	5.2145	.3755	-.3749
7.59	2230.9	1.9595	.1408	-.0198	5.2219	.3752	-.3746
7.60	2275.0	1.9609	.1406	-.0197	5.2293	.3750	-.3744
7.61	2320.1	1.9623	.1404	-.0197	5.2367	.3747	-.3741
7.62	2365.1	1.9637	.1402	-.0196	5.2442	.3745	-.3739
7.63	2413.0	1.9651	.1400	-.0196	5.2516	.3742	-.3736
7.64	2460.[illegible]	1.9665	.1398	-.0195	5.2590	.3739	-.3733
7.65	2509.2	1.9679	.1396	-.0195	5.2664	.3737	-.3731
7.66	2559.7	1.9693	.1394	-.0194	5.2738	.3734	-.3728
7.67	2610.7	1.9707	.1392	-.0194	5.2812	.3732	-.3726
7.68	2662.5	1.9721	.1391	-.0193	5.2886	.3729	-.3723
7.69	2715.7	1.9735	.1389	-.0193	5.2960	.3726	-.3720
7.70	2769.3	1.9749	.1387	-.0192	5.3034	.3724	-.3718
7.71	2825.1	1.9763	.1385	-.0191	5.3108	.3721	-.3715
7.72	2881.5	1.9777	.1383	-.0191	5.3182	.3719	-.3713
7.73	2939.1	1.9790	.1381	-.0190	5.3256	.3716	-.3710
7.74	2997.8	1.9804	.1379	-.0190	5.3330	.3714	-.3708
7.75	3057.6	1.9818	.1377	-.0189	5.3403	.3711	-.3705
7.76	3118.0	1.9832	.1375	-.0189	5.3477	.3708	-.3703
7.77	3181.5	1.9845	.1373	-.0188	5.3551	.3706	-.3700
7.78	3245.3	1.9859	.1371	-.0188	5.3625	.3703	-.3698
7.79	3310.4	1.9873	.1370	-.0187	5.3699	.3701	-.3695
7.80	3376.9	1.9887	.1368	-.0187	5.3772	.3698	-.3693
7.81	3444.[illegible]	1.9900	.1366	-.0186	5.3846	.3696	-.3690
7.82	3514.0	1.9914	.1364	-.0186	5.3920	.3693	-.3688
7.83	3584.7	1.9927	.1362	-.0185	5.3993	.3691	-.3685
7.84	3655.9	1.9941	.1360	-.0185	5.4067	.3688	-.3683
7.85	3730.6	1.9955	.1358	-.0184	5.4140	.3686	-.3680
7.86	3805.9	1.9968	.1357	-.0184	5.4214	.3683	-.3678
7.87	3882.6	1.9982	.1355	-.0183	5.4288	.3681	-.3675
7.88	3961.0	1.9995	.1353	-.0183	5.4361	.3678	-.3673
7.89	4041.0	2.0009	.1351	-.0182	5.4435	.3676	-.3670
7.90	4122.7	2.0022	.1349	-.0182	5.4508	.3673	-.3668
7.91	4206.1	2.0036	.1347	-.0181	5.4591	.3671	-.3665
7.92	4291.3	2.0049	.1346	-.0181	5.4655	.3668	-.3663
7.93	4378.2	2.0063	.1344	-.0180	5.4728	.3666	-.3660
7.94	4465.[illegible]	2.0076	.1342	-.0180	5.4802	.3663	-.3658
7.95	4557.6	2.0090	.1340	-.0179	5.4875	.3661	-.3656
7.96	4650.1	2.0103	.1339	-.0179	5.4948	.3659	-.3653
7.97	4744.5	2.0116	.1337	-.0178	5.5022	.3656	-.3651
7.98	4841.0	2.0130	.1335	-.0178	5.5095	.3654	-.3649
7.99	4937.4	2.0143	.1333	-.0177	5.5168	.3651	-.3646

X	GAMMA	PS	PS1	PS2	PS/VPS1	VPS1	PS2/PS1**[illegible]
8.00	5040.9	2.0156	.1331	-.0177	5.5241	.3649	-.3643
8.01	5142.6	2.0170	.1330	-.0177	5.5314	.3646	-.3641
8.02	5247.[illegible]	2.01[illegible]3	.132[illegible]	-.0176	5.5388	.3644	-.3639
8.03	5354.[illegible]	2.0196	.132[illegible]	-.0176	5.5461	.3642	-.3636
8.04	5463.7	2.0210	.1324	-.0175	5.5534	.3639	-.3634
8.05	5575.3	2.0223	.1323	-.0175	5.5607	.3637	-.3631
8.06	568[illegible].3	2.0236	.1321	-.0174	5.5680	.3634	-.3629
8.07	[illegible]	2.0249	.1319	-.0174	5.5753	.3632	-.3627
8.08	5924.4	2.0262	.1317	-.0173	5.5826	.3630	-.3624
8.09	6045.7	2.0276	.1316	-.0173	5.5899	.3627	-.3622
8.10	6169.6	2.0289	.1314	-.0172	5.5972	.3625	-.3620
8.11	6296.1	2.0302	.1312	-.0172	5.6045	.3622	-.3617
8.12	6425.2	2.0315	.1310	-.0171	5.6118	.3620	-.3615
8.13	6557.1	2.0328	.1309	-.0171	5.6191	.3618	-.3613
8.14	6691.8	2.0341	.1307	-.0171	5.6264	.3615	-.3610
8.15	6829.4	2.0354	.1305	-.0170	5.6337	.3613	-.3608
8.16	6969.[illegible]	2.0367	.1304	-.0170	5.6410	.3611	-.3606
8.17	7113.3	2.0380	.1302	-.0169	5.6482	.3608	-.3603
8.18	7259.[illegible]	2.0393	.1300	-.0169	5.6555	.3606	-.3601
8.19	7409.5	2.0406	.1299	-.0168	5.6628	.3604	-.3599
8.20	7562.3	2.0419	.1297	-.0168	5.6701	.3601	-.3596
8.21	7719.3	2.0432	.1295	-.0168	5.6773	.3599	-.3594
8.22	7877.7	2.0445	.1294	-.0167	5.6846	.3597	-.3592
8.23	8040.5	2.0458	.1292	-.0167	5.6919	.3594	-.3589
8.24	8206.7	2.0471	.1290	-.0166	5.6991	.3592	-.3587
8.25	8376.5	2.0484	.1289	-.0166	5.7064	.3590	-.3585
8.26	8549.[illegible]	2.0497	.1287	-.0165	5.7137	.3587	-.3582
8.27	8727.0	2.0510	.1285	-.0165	5.7209	.3585	-.3580
8.28	8907.[illegible]	2.0522	.1284	-.0165	5.7282	.3583	-.3578
8.29	9092.7	2.0535	.1282	-.0164	5.7354	.3580	-.3576
8.30	9281.4	2.0548	.1280	-.0164	5.7427	.3578	-.3573
8.31	9474.1	2.0561	.1279	-.0163	5.7499	.3576	-.3571
8.32	9671.0	2.0574	.1277	-.0163	5.7572	.3574	-.3569
8.33	9872.1	2.0586	.1275	-.0162	5.7644	.3571	-.3567
8.34	10077.5	2.0599	.1274	-.0162	5.7717	.3569	-.3564
8.35	10287.3	2.0612	.1272	-.0162	5.7789	.3567	-.3562
8.36	10501.6	2.0625	.1271	-.0161	5.7861	.3564	-.3560
8.37	10720.5	2.0637	.1269	-.0161	5.7934	.3562	-.3557
8.38	10944.1	2.0650	.1267	-.0160	5.8006	.3560	-.3555
8.39	11172.5	2.0663	.1266	-.0160	5.8078	.3558	-.3553
8.40	11405.8	2.0675	.1264	-.0160	5.8151	.3555	-.3551
8.41	11644.2	2.0688	.1263	-.0159	5.8223	.3553	-.3549
8.42	11[illegible]7.7	2.0701	.1261	-.0159	5.8295	.3551	-.3546
8.43	12135.4	2.0713	.1259	-.0159	5.8367	.3549	-.3544
8.44	12390.5	2.0726	.1258	-.0158	5.8439	.3547	-.3542
8.45	12650.0	2.0738	.1256	-.0158	5.8512	.3544	-.3540
8.46	[illegible]	2.0751	.1255	-.0157	5.8584	.3542	-.3537
8.47	13186.1	2.0763	.1253	-.0157	5.8656	.3540	-.3535
8.48	13462.[illegible]	2.0775	.1252	-.0156	5.8728	.3538	-.3533
8.49	13745.5	2.0788	.1250	-.0156	5.8800	.3535	-.3531

X	GAMMA	PS	PS1	PS2	PS/VPS1	VPS1	PS2/PS1**3/2
8.50	14034.4	2.0801	.1249	-.0156	5.8872	.3533	-.3529
8.51	1432[illegible].4	2.0813	.1247	-.0155	5.8944	.3531	-.3527
8.52	14630.	2.0826	.1245	-.0155	5.9016	.3529	-.3524
8.53	[illegible]	2.0838	.1244	-.0154	5.9088	.3527	-.3522
8.54	15253.[illegible]	2.0851	.1242	-.0154	5.9160	.3524	-.3520
8.55	15575.0	2.0863	.1241	-.0154	5.9232	.3522	-.3518
8.56	15903.5	2.0876	.1239	-.0153	5.9304	.3520	-.3516
8.57	16237.1	2.0888	.1238	-.0153	5.9376	.3518	-.3513
8.58	16581.[illegible]	2.0900	.1236	-.0153	5.9448	.3516	-.3511
8.59	16932.2	2.0913	.1235	-.0152	5.9519	.3514	-.3509
8.60	17290.2	2.0925	.1233	-.0152	5.9591	.3511	-.3507
8.61	17655.[illegible]	2.0937	.1231	-.0151	5.9663	.3509	-.3505
8.62	18029.6	2.0950	.1230	-.0151	5.9735	.3507	-.3503
8.63	18411.4	2.0962	.1228	-.0151	5.9806	.3505	-.3501
8.64	18801.5	2.0974	.1227	-.0150	5.9878	.3503	-.3498
8.65	19200.1	2.0986	.1225	-.0150	5.9950	.3501	-.3496
8.66	19607.5	2.0999	.1224	-.0150	6.0022	.3499	-.3494
8.67	20023.7	2.1011	.1222	-.0149	6.0093	.3496	-.3492
8.68	2044[illegible].0	2.1023	.1221	-.0149	6.0165	.3494	-.3490
8.69	20883.5	2.1035	.1219	-.0149	6.0237	.3492	-.3488
8.70	21327.6	2.1048	.1218	-.0148	6.0308	.3490	-.3486
8.71	21781.4	2.1060	.1217	-.0148	6.0380	.3488	-.3484
8.72	22245.1	2.1072	.1215	-.0147	6.0451	.3486	-.3482
8.73	22718.[illegible]	2.1084	.1214	-.0147	6.0523	.3484	-.3479
8.74	23203.2	2.1096	.1212	-.0147	6.0594	.3482	-.3477
8.75	23698.0	2.1108	.1211	-.0146	6.0666	.3479	-.3475
8.76	24203.7	2.1120	.1209	-.0146	6.0737	.3477	-.3473
8.77	24720.5	2.1132	.1208	-.0146	6.0809	.3475	-.3471
8.78	2524[illegible].6	2.1144	.1206	-.0145	6.0880	.3473	-.3469
8.79	25788.3	2.1157	.1205	-.0145	6.0951	.3471	-.3467
8.80	26339.9	2.1169	.1203	-.0145	6.1023	.3469	-.3465
8.81	26903.5	2.1181	.1202	-.0144	6.1094	.3467	-.3463
8.82	27477.6	2.1193	.1200	-.0144	6.1166	.3465	-.3461
8.83	28062.4	2.1205	.1199	-.0144	6.1237	.3463	-.3459
8.84	28670.1	2.1217	.1198	-.0143	6.1308	.3461	-.3457
8.85	29285.0	2.1229	.1196	-.0143	6.1379	.3459	-.3454
8.86	29913.5	2.1241	.1195	-.0143	6.1451	.3457	-.3452
8.87	30555.8	2.1252	.1193	-.0142	6.1522	.3454	-.3450
8.88	31212.4	2.1264	.1192	-.0142	6.1593	.3452	-.3448
8.89	31863.4	2.1276	.1190	-.0142	6.1664	.3450	-.3446
8.90	32569.2	2.1288	.1189	-.0141	6.1735	.3448	-.3444
8.91	33270.2	2.1300	.1188	-.0141	6.1806	.3446	-.3442
8.92	33985.7	2.1312	.1186	-.0141	6.1877	.3444	-.3440
8.93	3471[illegible].0	2.1324	.1185	-.0140	6.1[illegible]	.3442	-.3438
8.94	35467.5	2.1336	.1183	-.0140	6.2020	.3440	-.3436
8.95	36232.5	2.1347	.1182	-.0140	6.2091	.3438	-.3434
8.96	37014.6	2.1359	.1181	-.0139	6.2162	.3436	-.3432
8.97	37813.[illegible]	2.1371	.1179	-.0139	6.2233	.3434	-.3430
8.98	38630.[illegible]	2.1383	.1178	-.0139	6.2304	.3432	-.3428
8.99	39466.1	2.1395	.1177	-.0138	6.2375	.3430	-.3426

Y	GAMMA	PS	PS1	PS2	PS/VPS1	VPS1	PS2/PS1**3/2
9.00	4031[illegible]	2.1406	.1175	-.0133	6.2446	.3428	-.3424
9.01	411[illegible]	2.1417	.1174	-.0135	6.2517	.3426	-.3422
9.02	42[illegible]	2.1430	.1172	-.0137	6.2588	.3424	-.3420
9.03	42[illegible]	2.1442	.1171	-.0137	6.2659	.3422	-.3419
9.04	43[illegible].5	2.1453	.1170	-.0137	6.272[illegible]	.3420	-.3416
9.05	44881.3	2.1465	.1169	-.0136	6.2800	.3418	-.3414
9.06	45[illegible]55.4	2.1477	.1167	-.0136	6.2871	.3416	-.3412
9.07	46[illegible]51.1	2.14[illegible]	.1166	-.0136	6.2942	.3414	-.3410
9.08	47[illegible].0	2.1500	.1164	-.0135	6.3012	.3412	-.3408
9.09	4[illegible].6	2.1512	.1163	-.0135	6.3083	.3410	-.3406
9.10	49973.4	2.1523	.1161	-.0135	6.3154	.3408	-.3404
9.11	51061.0	2.1535	.1160	-.0134	6.3225	.3406	-.3402
9.12	52172.[illegible]	2.1546	.1159	-.0134	6.3295	.3404	-.3400
9.13	5330[illegible].4	2.1558	.1157	-.0134	6.3366	.3402	-.3398
9.14	54471.5	2.1570	.1156	-.0134	6.3437	.3400	-.3396
9.15	55659.5	2.1581	.1155	-.0133	6.3507	.3398	-.3394
9.16	56874.1	2.1593	.1153	-.0133	6.3578	.3396	-.3393
9.17	58115.[illegible]	2.1604	.1152	-.0133	6.3648	.3394	-.3391
9.18	59385.4	2.1616	.1151	-.0132	6.3719	.3392	-.3389
9.19	60683.4	2.1627	.1149	-.0132	6.3790	.3390	-.3387
9.20	62010.4	2.1639	.1148	-.0132	6.3860	.3388	-.3385
9.21	63367.2	2.1650	.1147	-.0131	6.3931	.3387	-.3383
9.22	64754.5	2.1662	.1146	-.0131	6.4001	.3385	-.3381
9.23	66172.[illegible]	2.1673	.1144	-.0131	6.4071	.3383	-.3379
9.24	67623.1	2.1685	.1143	-.0130	6.4142	.3381	-.3377
9.25	69105.8	2.1696	.1142	-.0130	6.4212	.3379	-.3375
9.26	70622.0	2.1707	.1140	-.0130	6.4283	.3377	-.3373
9.27	72172.1	2.1719	.1139	-.0130	6.4353	.3375	-.3371
9.28	73757.2	2.1730	.1138	-.0129	6.4423	.3373	-.3369
9.29	75377.9	2.1742	.1136	-.0129	6.4494	.3371	-.3368
9.30	77035.1	2.1753	.1135	-.0129	6.4564	.3369	-.3366
9.31	78729.7	2.1764	.1134	-.0128	6.4634	.3367	-.3364
9.32	80462.4	2.1776	.1133	-.0128	6.4705	.3365	-.3362
9.33	82234.2	2.1787	.1131	-.0128	6.4775	.3363	-.3360
9.34	84045.9	2.1798	.1130	-.0128	6.4845	.3362	-.3358
9.35	85898.6	2.1809	.1129	-.0127	6.4915	.3360	-.3356
9.36	87793.1	2.1821	.1127	-.0127	6.4985	.3358	-.3354
9.37	89730.3	2.1832	.1126	-.0127	6.5056	.3356	-.3352
9.38	91711.4	2.1843	.1125	-.0126	6.5126	.3354	-.3351
9.39	93737.2	2.1855	.1124	-.0126	6.5196	.3352	-.3349
9.40	9580[illegible]	2.1866	.1122	-.0126	6.5266	.3350	-.3347
9.41	97[illegible]27.5	2.1877	.1121	-.0126	6.5336	.3348	-.3345
9.42	100[illegible]4.0	2.188[illegible]	.1120	-.0125	6.5406	.3347	-.3343
9.43	10230[illegible]	2.1[illegible]	.1119	-.0125	6.5476	.3345	-.3341
9.44	104575.4	2.1911	.1117	-.0125	6.5546	.3343	-.3339
9.45	106[illegible]2.6	2.1922	.1116	-.0124	6.5616	.3341	-.3337
9.46	10[illegible]262.3	2.1933	.1115	-.0124	6.5686	.3339	-.3336
9.47	1116[illegible]	2.1944	.1114	-.0124	6.5756	.3337	-.3334
9.48	114164.4	2.1955	.1112	-.0124	6.5826	.3335	-.3332
9.49	11669[illegible].3	2.1966	.1111	-.0123	6.5896	.3333	-.3330

X	GAMMA	PS	PS1	PS2	PS/VPS1	VPS1	PS2/PS1**3/2
9.50	119291.7	2.1[illegible]77	.1110	-.0123	6.5966	.3332	-.332[illegible]
9.51	121[illegible]43.1	2.1[illegible]	.110[illegible]	-.0123	6.6036	.3330	-.3326
9.52	124654.[illegible]	2.2000	.110[illegible]	-.0123	6.6106	.3328	-.3325
9.53	12[illegible]42[illegible].3	2.2011	.110[illegible]	-.0122	6.6175	.3326	-.3323
9.54	130[illegible]64.[illegible]	2.2022	.1105	-.0122	6.6245	.3324	-.3321
9.55	133166.1	2.2033	.1104	-.0122	6.6315	.3322	-.3319
9.56	136133.4	2.2044	.1103	-.0121	6.6385	.3321	-.3317
9.57	13[illegible]16[illegible].4	2.2055	.1101	-.0121	6.6455	.3319	-.3315
9.58	142272.[illegible]	2.2066	.1100	-.0121	6.6524	.3317	-.3314
9.59	145447.6	2.2077	.1099	-.0121	6.6594	.3315	-.3312
9.60	148695.2	2.2088	.1098	-.0120	6.6664	.3313	-.3310
9.61	152016.3	2.2099	.1097	-.0120	6.6733	.3311	-.3308
9.62	155414.5	2.2110	.1095	-.0120	6.6803	.3310	-.3306
9.63	1588[illegible].[illegible]	2.2121	.1094	-.0120	6.6873	.3308	-.3305
9.64	162444.6	2.2132	.1093	-.0119	6.6942	.3306	-.3303
9.65	166080.9	2.2143	.1092	-.0119	6.7012	.3304	-.3301
9.66	169800.1	2.2153	.1091	-.0119	6.7082	.3302	-.3299
9.67	173604.7	2.2164	.1089	-.0119	6.7151	.3301	-.3297
9.68	177496.5	2.2175	.1088	-.0118	6.7221	.3299	-.3296
9.69	181477.4	2.2186	.1087	-.0118	6.7290	.3297	-.3294
9.70	185549.7	2.2197	.1086	-.0118	6.7360	.3295	-.3292
9.71	189715.4	2.2208	.1085	-.0118	6.7429	.3294	-.3290
9.72	193976.8	2.2219	.1084	-.0117	6.7499	.3292	-.3289
9.73	198335.9	2.2229	.1082	-.0117	6.7568	.3290	-.3287
9.74	202795.3	2.2240	.1081	-.0117	6.7637	.3288	-.3285
9.75	207357.2	2.2251	.1080	-.0117	6.7707	.3286	-.3283
9.76	212023.[illegible]	2.2262	.1079	-.0116	6.7776	.3285	-.3281
9.77	21679[illegible].1	2.2273	.1078	-.0116	6.7846	.3283	-.3280
9.78	221662.2	2.2283	.1077	-.0116	6.7915	.3281	-.3278
9.79	226678.7	2.2294	.1075	-.0116	6.7984	.3279	-.3276
9.80	231790.3	2.2305	.1074	-.0115	6.8054	.3278	-.3274
9.81	23701[illegible].7	2.2316	.1073	-.0115	6.8123	.3276	-.3273
9.82	24236[illegible].7	2.2326	.1072	-.0115	6.8192	.3274	-.3271
9.83	247843.2	2.2337	.1071	-.0115	6.8261	.3272	-.3269
9.84	253442.8	2.2348	.1070	-.0114	6.8331	.3271	-.3267
9.85	259172.0	2.2359	.1069	-.0114	6.8400	.3269	-.3266
9.86	265033.3	2.2369	.1067	-.0114	6.8469	.3267	-.3264
9.87	271030.1	2.2380	.1066	-.0114	6.8539	.3265	-.3262
9.88	27716[illegible].7	2.2391	.1065	-.0113	6.8607	.3264	-.3261
9.89	283443.1	2.2401	.1064	-.0113	6.8676	.3262	-.3259
9.90	289465.6	2.2412	.1063	-.0113	6.8746	.3260	-.3257
9.91	2[illegible]6437.0	2.2422	.1062	-.0113	6.8815	.3258	-.3255
9.92	303160.[illegible]	2.2433	.1061	-.0112	6.8884	.3257	-.3254
9.93	31003[illegible].7	2.2444	.1059	-.0112	6.8953	.3255	-.3252
9.94	31707[illegible].5	2.2454	.105[illegible]	-.0112	6.9022	.3253	-.3250
9.95	324280.[illegible]	2.2465	.1057	-.0112	6.9091	.3251	-.3248
9.96	33164[illegible].[illegible]	2.2475	.1056	-.0111	6.9160	.3250	-.3247
9.97	33[illegible]18[illegible].[illegible]	2.2486	.1055	-.0111	6.9229	.3248	-.3245
9.98	34[illegible]04.[illegible]	2.2496	.1054	-.0111	6.92[illegible]	.3246	-.3243
9.99	35475[illegible].4	2.2507	.1053	-.0111	6.9367	.3245	-.3242
10.00	362[illegible]77.3	2.2518	.1052	-.0110	6.9436	.3243	-.3240
x	→	$\ln x$	$1/x$	$-1/x^2$	$\sqrt{x}\ln x$	$1/\sqrt{x}$	$-1/\sqrt{x}$

APPENDIX B
EMPIRICAL DATA FROM FIELD MEASUREMENTS

Statistical long term distributions from full scale measurements on T/T Esso Bonn /14/.

Particulars:

Length overall	347.800 M
Length on summer LWL	337.861 "
Length between perpendiculars	329.200 "
Breadth moulded	51.800 "
Depth moulded	25.600 "

Drafts:

Tropical freshwater	20.913 "
Freshwater	20.498 "
Tropical	20.458 "
Summer	20.043 "
Winter	19.628 "

Weights:

Light weight	36.063 T
Light weight V.C.G. Above Base	14.30 M
Light weight LCG FWD of AP	150.33 "
Load displacement summer	292.758 T
" " tropical	299.493 T

Tonnage International:

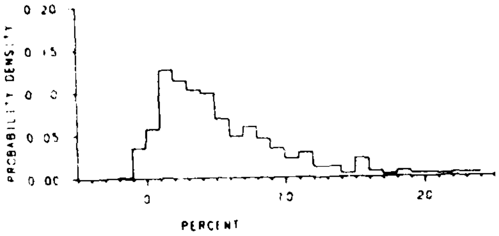
Gross	126.192,23
Net	99.621,46

Section modulus

Top midship section modulus	76.23 M^3
Bottom midship section modulus	77.27 M^3
Midship second moment of steel area	982.38 M^4

Main data for A.G. WESER yard No. 1388
T.T. "ESSO BONN"

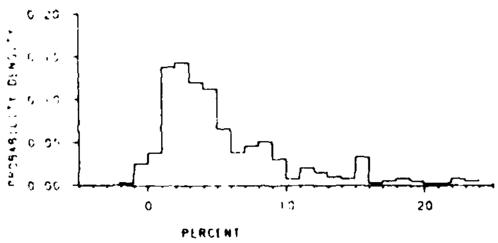
DEVIATION IN RMS. 1-SQRT(RMSB**2+RMSS**2)/RMST
ALL LOADING CONDITIONS



STATISTICAL PARAMETERS

NUMBER OF SAMPLE VALUES	725
ARITHMETIC MEAN VALUE, M1	8.172+00
STANDARD DEVIATION, S	1.201+01
COEFFICIENT OF VARIATION, M1/S	6.804-01
COEFFICIENT OF SKEWNESS, K3/S**3	5.771+00
COEFFICIENT OF EXCESS, K4/S**4	1.706+01
SECOND CENTRAL MOMENT, C2	1.443+02
THIRD CENTRAL MOMENT, C3	6.533+03
FOURTH CENTRAL MOMENT, C4	4.174+05
FOURTH CUMULANT, K4=(C4-3*C2**2)	3.550+05
SECOND MOMENT ABOUT ZERO, M2	2.101+02
THIRD MOMENT ABOUT ZERO, M3	1.057+04
FOURTH MOMENT ABOUT ZERO, M4	6.895+05
MINIMUM VALUE	-1.254+00
MAXIMUM VALUE	9.441+01

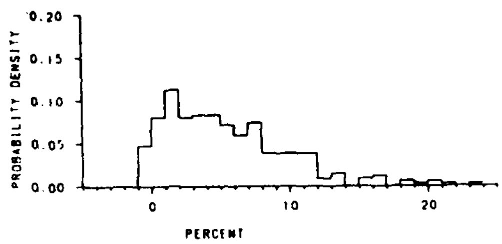
DEVIATION IN RMS. 1-SQRT(RMSB**2+RMSS**2)/RMST
BALLAST CONDITIONS



STATISTICAL PARAMETERS

NUMBER OF SAMPLE VALUES	389
ARITHMETIC MEAN VALUE, M1	7.533+00
STANDARD DEVIATION, S	1.088+01
COEFFICIENT OF VARIATION, M1/S	6.922-01
COEFFICIENT OF SKEWNESS, K3/S**3	4.187+00
COEFFICIENT OF EXCESS, K4/S**4	2.243+01
SECOND CENTRAL MOMENT, C2	1.1[illegible]+02
THIRD CENTRAL MOMENT, C3	5.394+03
FOURTH CENTRAL MOMENT, C4	3.565+05
FOURTH CUMULANT, K4=(C4-3*C2**2)	3.144+05
SECOND MOMENT ABOUT ZERO, M2	1.748+02
THIRD MOMENT ABOUT ZERO, M3	8.435+03
FOURTH MOMENT ABOUT ZERO, M4	5.569+05
MINIMUM VALUE	-1.254+00
MAXIMUM VALUE	9.441+01

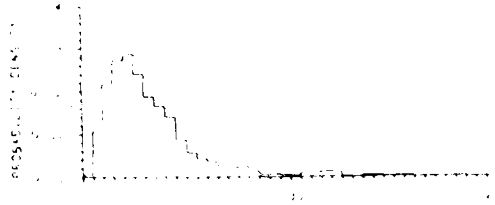
DEVIATION IN RMS. 1-SQRT(RMSB**2+RMSS**2)/RMST
FULL LOAD CONDITIONS



STATISTICAL PARAMETERS

NUMBER OF SAMPLE VALUES	336
ARITHMETIC MEAN VALUE, M1	8.913+00
STANDARD DEVIATION, S	1.318+01
COEFFICIENT OF VARIATION, M1/S	6.765-01
COEFFICIENT OF SKEWNESS, K3/S**3	3.414+00
COEFFICIENT OF EXCESS, K4/S**4	1.311+01
SECOND CENTRAL MOMENT, C2	1.735+02
THIRD CENTRAL MOMENT, C3	7.809+03
FOURTH CENTRAL MOMENT, C4	4.855+05
FOURTH CUMULANT, K4=(C4-3*C2**2)	3.951+05
SECOND MOMENT ABOUT ZERO, M2	2.529+02
THIRD MOMENT ABOUT ZERO, M3	1.305+04
FOURTH MOMENT ABOUT ZERO, M4	9.431+05
MINIMUM VALUE	-7.204-01
MAXIMUM VALUE	8.469+01

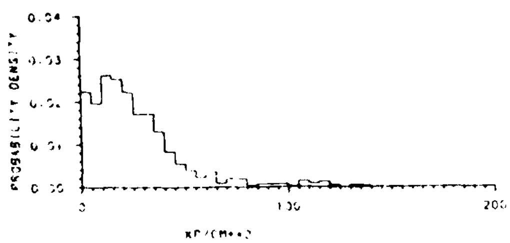
RMS FOR TOTAL STRESS
ALL LOADING CONDITIONS



STATISTICAL PARAMETERS

NUMBER OF SAMPLE VALUES	1116
ARITHMETIC MEAN VALUE, M1	3.324+01
STANDARD DEVIATION, S	2.290+01
COEFFICIENT OF VARIATION, M1/S	1.451+00
COEFFICIENT OF SKEWNESS, K3/S**3	2.110+00
COEFFICIENT OF EXCESS, K4/S**4	5.773+00
SECOND CENTRAL MOMENT, C2	5.246+02
THIRD CENTRAL MOMENT, C3	2.535+04
FOURTH CENTRAL MOMENT, C4	2.414+06
FOURTH CUMULANT, K4=(C4-3*C2**2)	1.589+06
SECOND MOMENT ABOUT ZERO, M2	1.629+03
THIRD MOMENT ABOUT ZERO, M3	1.143+05
FOURTH MOMENT ABOUT ZERO, M4	1.044+07
MINIMUM VALUE	4.262+00
MAXIMUM VALUE	1.577+02

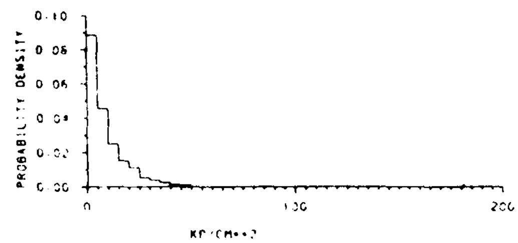
RMS FOR BENDING STRESS
ALL LOADING CONDITIONS



STATISTICAL PARAMETERS

NUMBER OF SAMPLE VALUES	1119
ARITHMETIC MEAN VALUE, M1	2.729+01
STANDARD DEVIATION, S	2.363+01
COEFFICIENT OF VARIATION, M1/S	1.155+00
COEFFICIENT OF SKEWNESS, K3/S**3	2.058+00
COEFFICIENT OF EXCESS, K4/S**4	5.403+00
SECOND CENTRAL MOMENT, C2	5.583+02
THIRD CENTRAL MOMENT, C3	2.715+04
FOURTH CENTRAL MOMENT, C4	2.619+06
FOURTH CUMULANT, K4=(C4-3*C2**2)	1.684+06
SECOND MOMENT ABOUT ZERO, M2	1.303+03
THIRD MOMENT ABOUT ZERO, M3	9.304+04
FOURTH MOMENT ABOUT ZERO, M4	8.608+06
MINIMUM VALUE	1.414+00
MAXIMUM VALUE	1.334+02

RMS FOR SPRINGING STRESS
ALL LOADING CONDITIONS

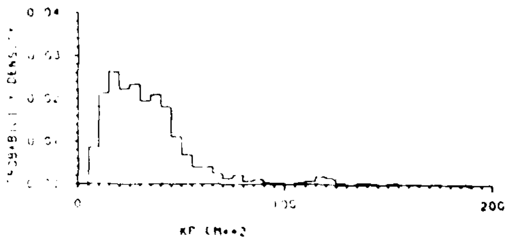


STATISTICAL PARAMETERS

NUMBER OF SAMPLE VALUES	1119
ARITHMETIC MEAN VALUE, M1	9.535+00
STANDARD DEVIATION, S	9.353+00
COEFFICIENT OF VARIATION, M1/S	1.019+00
COEFFICIENT OF SKEWNESS, K3/S**3	1.842+00
COEFFICIENT OF EXCESS, K4/S**4	4.052+00
SECOND CENTRAL MOMENT, C2	8.749+01
THIRD CENTRAL MOMENT, C3	1.508+03
FOURTH CENTRAL MOMENT, C4	5.398+04
FOURTH CUMULANT, K4=(C4-3*C2**2)	3.102+04
SECOND MOMENT ABOUT ZERO, M2	1.783+02
THIRD MOMENT ABOUT ZERO, M3	4.869+03
FOURTH MOMENT ABOUT ZERO, M4	1.670+05
MINIMUM VALUE	7.070-01
MAXIMUM VALUE	6.858+01

RMS FOR TOTAL STRESS

BALLAST CONDITIONS

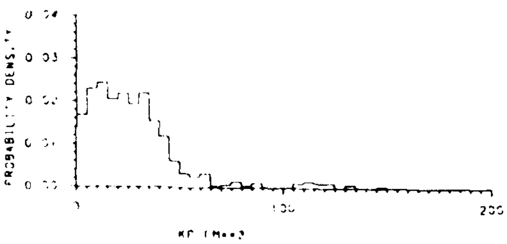


STATISTICAL PARAMETERS

NUMBER OF SAMPLE VALUES	557
ARITHMETIC MEAN VALUE, M1	3.430+01
STANDARD DEVIATION, S	2.239+01
COEFFICIENT OF VARIATION, M1/S	1.532+00
COEFFICIENT OF SKEWNESS, K3/S**3	2.092+00
COEFFICIENT OF EXCESS, K4/S**4	5.959+00
SECOND CENTRAL MOMENT, C2	5.013+02
THIRD CENTRAL MOMENT, C3	2.348+04
FOURTH CENTRAL MOMENT, C4	2.251+06
FOURTH CUMULANT, K4=(C4-3*C2**2)	1.498+06
SECOND MOMENT ABOUT ZERO, M2	1.677+03
THIRD MOMENT ABOUT ZERO, M3	1.151+05
FOURTH MOMENT ABOUT ZERO, M4	1.035+07
MINIMUM VALUE	7.070+00
MAXIMUM VALUE	1.541+02

RMS FOR BENDING STRESS

BALLAST CONDITIONS

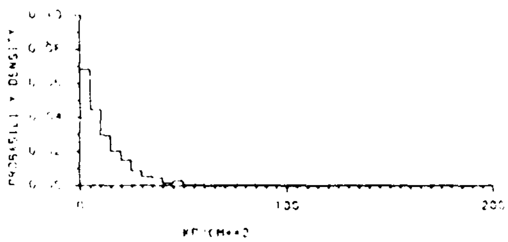


STATISTICAL PARAMETERS

NUMBER OF SAMPLE VALUES	557
ARITHMETIC MEAN VALUE, M1	2.763+01
STANDARD DEVIATION, S	2.244+01
COEFFICIENT OF VARIATION, M1/S	1.232+00
COEFFICIENT OF SKEWNESS, K3/S**3	2.164+00
COEFFICIENT OF EXCESS, K4/S**4	6.480+00
SECOND CENTRAL MOMENT, C2	5.036+02
THIRD CENTRAL MOMENT, C3	2.446+04
FOURTH CENTRAL MOMENT, C4	2.404+06
FOURTH CUMULANT, K4=(C4-3*C2**2)	1.643+06
SECOND MOMENT ABOUT ZERO, M2	1.267+03
THIRD MOMENT ABOUT ZERO, M3	8.713+04
FOURTH MOMENT ABOUT ZERO, M4	7.961+06
MINIMUM VALUE	1.414+00
MAXIMUM VALUE	1.471+02

RMS FOR SPRINGING STRESS

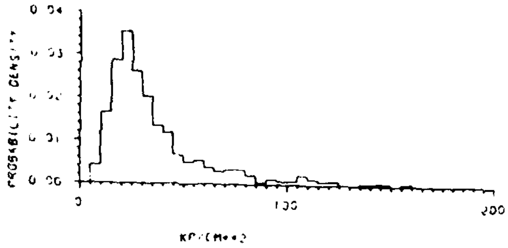
BALLAST CONDITIONS



STATISTICAL PARAMETERS

NUMBER OF SAMPLE VALUES	557
ARITHMETIC MEAN VALUE, M1	1.188+01
STANDARD DEVIATION, S	1.069+01
COEFFICIENT OF VARIATION, M1/S	1.111+00
COEFFICIENT OF SKEWNESS, K3/S**3	1.537+00
COEFFICIENT OF EXCESS, K4/S**4	2.631+00
SECOND CENTRAL MOMENT, C2	1.144+02
THIRD CENTRAL MOMENT, C3	1.880+03
FOURTH CENTRAL MOMENT, C4	7.366+04
FOURTH CUMULANT, K4=(C4-3*C2**2)	3.441+04
SECOND MOMENT ABOUT ZERO, M2	2.555+02
THIRD MOMENT ABOUT ZERO, M3	7.615+03
FOURTH MOMENT ABOUT ZERO, M4	2.783+05
MINIMUM VALUE	7.070-01
MAXIMUM VALUE	6.090+01

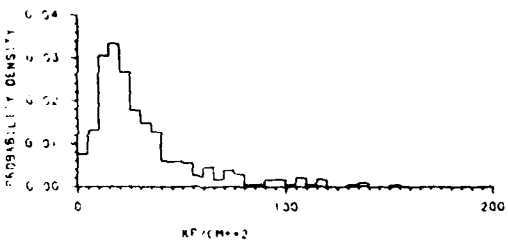
RMS FOR TOTAL STRESS
FULL LOAD CONDITIONS



STATISTICAL PARAMETERS

NUMBER OF SAMPLE VALUES	473
ARITHMETIC MEAN VALUE, M1	3.567+01
STANDARD DEVIATION, S	2.437+01
COEFFICIENT OF VARIATION, M1/S	1.463+00
COEFFICIENT OF SKEWNESS, K3/S**3	2.021+00
COEFFICIENT OF EXCESS, K4/S**4	4.688+00
SECOND CENTRAL MOMENT, C2	5.941+02
THIRD CENTRAL MOMENT, C3	2.927+04
FOURTH CENTRAL MOMENT, C4	2.714+06
FOURTH CUMULANT, K4=(C4-3*C2**2)	1.6[illegible]+06
SECOND MOMENT ABOUT ZERO, M2	1.865+03
THIRD MOMENT ABOUT ZERO, M3	1.378+05
FOURTH MOMENT ABOUT ZERO, M4	1.297+07
MINIMUM VALUE	7.777+00
MAXIMUM VALUE	1.577+02

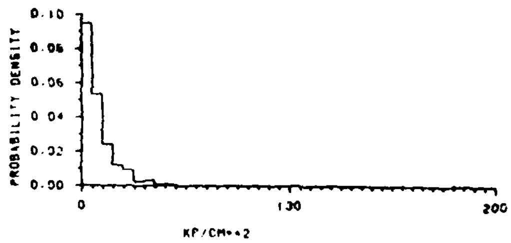
RMS FOR BENDING STRESS
FULL LOAD CONDITIONS



STATISTICAL PARAMETERS

NUMBER OF SAMPLE VALUES	473
ARITHMETIC MEAN VALUE, M1	3.094+01
STANDARD DEVIATION, S	2.493+01
COEFFICIENT OF VARIATION, M1/S	1.242+00
COEFFICIENT OF SKEWNESS, K3/S**3	1.957+00
COEFFICIENT OF EXCESS, K4/S**4	4.260+00
SECOND CENTRAL MOMENT, C2	6.216+02
THIRD CENTRAL MOMENT, C3	3.033+04
FOURTH CENTRAL MOMENT, C4	2.805+06
FOURTH CUMULANT, K4=(C4-3*C2**2)	1.646+06
SECOND MOMENT ABOUT ZERO, M2	1.579+03
THIRD MOMENT ABOUT ZERO, M3	1.174+05
FOURTH MOMENT ABOUT ZERO, M4	1.099+07
MINIMUM VALUE	1.414+00
MAXIMUM VALUE	1.534+02

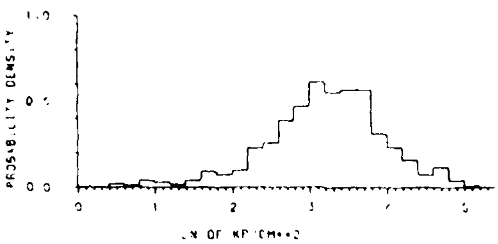
RMS FOR SPRINGING STRESS
FULL LOAD CONDITIONS



STATISTICAL PARAMETERS

NUMBER OF SAMPLE VALUES	473
ARITHMETIC MEAN VALUE, M1	8.051+00
STANDARD DEVIATION, S	7.164+00
COEFFICIENT OF VARIATION, M1/S	1.124+00
COEFFICIENT OF SKEWNESS, K3/S**3	1.823+00
COEFFICIENT OF EXCESS, K4/S**4	3.667+00
SECOND CENTRAL MOMENT, C2	5.132+01
THIRD CENTRAL MOMENT, C3	6.702+02
FOURTH CENTRAL MOMENT, C4	1.756+04
FOURTH CUMULANT, K4=(C4-3*C2**2)	9.656+03
SECOND MOMENT ABOUT ZERO, M2	1.160+02
THIRD MOMENT ABOUT ZERO, M3	2.423+03
FOURTH MOMENT ABOUT ZERO, M4	6.289+04
MINIMUM VALUE	1.414+00
MAXIMUM VALUE	6.242+01

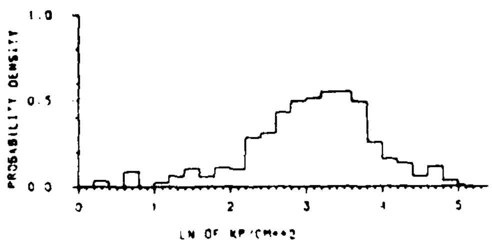
LN (RMS) FOR TOTAL STRESS
ALL LOADING CONDITIONS



STATISTICAL PARAMETERS

NUMBER OF SAMPLE VALUES	1030
ARITHMETIC MEAN VALUE, M1	3.203+00
STANDARD DEVIATION, S	7.602-01
COEFFICIENT OF VARIATION, M1/S	4.213+00
COEFFICIENT OF SKEWNESS, K3/S**3	-5.380-01
COEFFICIENT OF EXCESS, K4/S**4	9.691-01
SECOND CENTRAL MOMENT, C2	5.779-01
THIRD CENTRAL MOMENT, C3	-2.363-01
FOURTH CENTRAL MOMENT, C4	1.325+00
FOURTH CUMULANT, K4=(C4-3*C2**2)	3.236-01
SECOND MOMENT ABOUT ZERO, M2	1.083+01
THIRD MOMENT ABOUT ZERO, M3	3.816+01
FOURTH MOMENT ABOUT ZERO, M4	1.390+02
MINIMUM VALUE	4.581-01
MAXIMUM VALUE	5.053+00

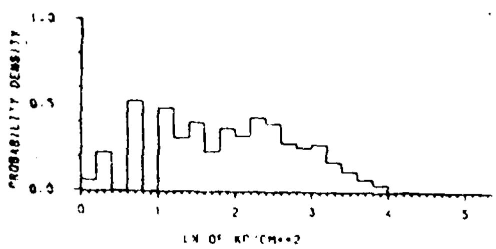
LN (RMS) FOR BENDING STRESS
ALL LOADING CONDITIONS



STATISTICAL PARAMETERS

NUMBER OF SAMPLE VALUES	1030
ARITHMETIC MEAN VALUE, M1	3.073+00
STANDARD DEVIATION, S	8.283-01
COEFFICIENT OF VARIATION, M1/S	3.710+00
COEFFICIENT OF SKEWNESS, K3/S**3	-6.054-01
COEFFICIENT OF EXCESS, K4/S**4	8.563-01
SECOND CENTRAL MOMENT, C2	6.861-01
THIRD CENTRAL MOMENT, C3	-3.440-01
FOURTH CENTRAL MOMENT, C4	1.815+00
FOURTH CUMULANT, K4=(C4-3*C2**2)	4.032-01
SECOND MOMENT ABOUT ZERO, M2	1.013+01
THIRD MOMENT ABOUT ZERO, M3	3.499+01
FOURTH MOMENT ABOUT ZERO, M4	1.256+02
MINIMUM VALUE	3.466-01
MAXIMUM VALUE	5.033+00

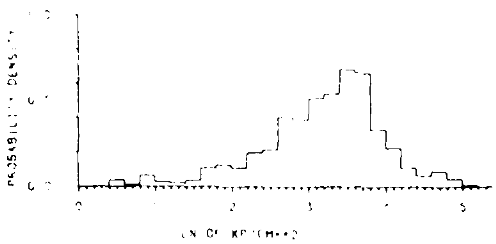
LN (RMS) FOR SPRINGING STRESS
ALL LOADING CONDITIONS



STATISTICAL PARAMETERS

NUMBER OF SAMPLE VALUES	1030
ARITHMETIC MEAN VALUE, M1	1.913+00
STANDARD DEVIATION, S	9.230-01
COEFFICIENT OF VARIATION, M1/S	2.070+00
COEFFICIENT OF SKEWNESS, K3/S**3	-2.906-02
COEFFICIENT OF EXCESS, K4/S**4	-7.339-01
SECOND CENTRAL MOMENT, C2	8.534-01
THIRD CENTRAL MOMENT, C3	-2.291-02
FOURTH CENTRAL MOMENT, C4	1.651+00
FOURTH CUMULANT, K4=(C4-3*C2**2)	-5.346-01
SECOND MOMENT ABOUT ZERO, M2	4.310+00
THIRD MOMENT ABOUT ZERO, M3	1.186+01
FOURTH MOMENT ABOUT ZERO, M4	3.356+01
MINIMUM VALUE	-3.466-01
MAXIMUM VALUE	4.228+00

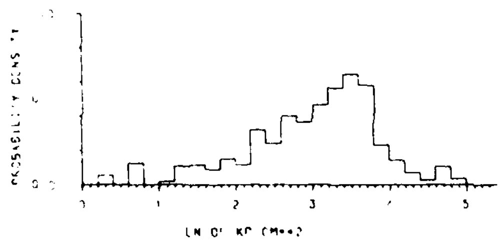
LN(RMS) FOR TOTAL STRESS
BALLAST CONDITIONS



STATISTICAL PARAMETERS

NUMBER OF SAMPLE VALUES	557
ARITHMETIC MEAN VALUE, M1	3.177+00
STANDARD DEVIATION, S	7.927-01
COEFFICIENT OF VARIATION, M1/S	4.007+00
COEFFICIENT OF SKEWNESS, K3/S**3	-7.552-01
COEFFICIENT OF EXCESS, K4/S**4	1.123+00
SECOND CENTRAL MOMENT, C2	6.285-01
THIRD CENTRAL MOMENT, C3	-3.763-01
FOURTH CENTRAL MOMENT, C4	1.628+00
FOURTH CUMULANT, K4=(C4-3*C2**2)	4.435-01
SECOND MOMENT ABOUT ZERO, M2	1.072+01
THIRD MOMENT ABOUT ZERO, M3	3.766+01
FOURTH MOMENT ABOUT ZERO, M4	1.367+02
MINIMUM VALUE	4.581-01
MAXIMUM VALUE	5.042+00

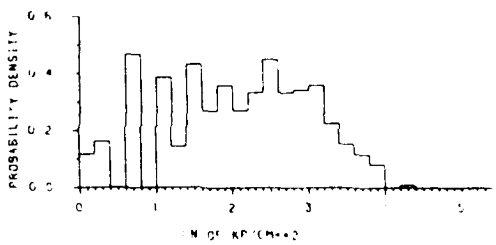
LN(RMS) FOR BENDING STRESS
BALLAST CONDITIONS



STATISTICAL PARAMETERS

NUMBER OF SAMPLE VALUES	557
ARITHMETIC MEAN VALUE, M1	3.0[illegible]+00
STANDARD DEVIATION, S	8.760-01
COEFFICIENT OF VARIATION, M1/S	3.424+00
COEFFICIENT OF SKEWNESS, K3/S**3	-7.228-01
COEFFICIENT OF EXCESS, K4/S**4	6.387-01
SECOND CENTRAL MOMENT, C2	7.674-01
THIRD CENTRAL MOMENT, C3	-4.859-01
FOURTH CENTRAL MOMENT, C4	2.143+00
FOURTH CUMULANT, K4=(C4-3*C2**2)	3.762-01
SECOND MOMENT ABOUT ZERO, M2	9.764+00
THIRD MOMENT ABOUT ZERO, M3	3.34[illegible]+01
FOURTH MOMENT ABOUT ZERO, M4	1.187+02
MINIMUM VALUE	3.4[illegible]-01
MAXIMUM VALUE	4.991+00

LN(RMS) FOR SPRINGING STRESS
BALLAST CONDITIONS

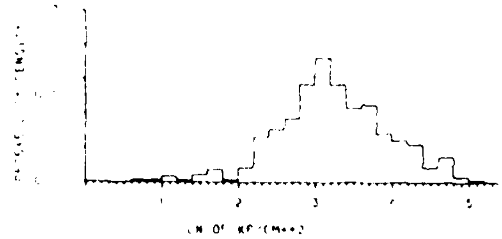


STATISTICAL PARAMETERS

NUMBER OF SAMPLE VALUES	557
ARITHMETIC MEAN VALUE, M1	2.054+00
STANDARD DEVIATION, S	9.791-01
COEFFICIENT OF VARIATION, M1/S	2.098+00
COEFFICIENT OF SKEWNESS, K3/S**3	-2.677-01
COEFFICIENT OF EXCESS, K4/S**4	-5.939-01
SECOND CENTRAL MOMENT, C2	9.586-01
THIRD CENTRAL MOMENT, C3	-2.512-01
FOURTH CENTRAL MOMENT, C4	2.211+00
FOURTH CUMULANT, K4=(C4-3*C2**2)	-5.657-01
SECOND MOMENT ABOUT ZERO, M2	5.176+00
THIRD MOMENT ABOUT ZERO, M3	1.431+01
FOURTH MOMENT ABOUT ZERO, M4	4.216+01
MINIMUM VALUE	-3.466-01
MAXIMUM VALUE	4.228+00

LN(RMS) FOR TOTAL STRESS

FULL LOAD CONDITIONS

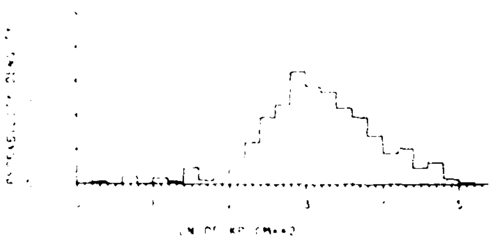


STATISTICAL PARAMETERS

NUMBER OF SAMPLE VALUES	473
ARITHMETIC MEAN VALUE, M1	3.233+00
STANDARD DEVIATION, S	7.195-01
COEFFICIENT OF VARIATION, M1/S	4.493+00
COEFFICIENT OF SKEWNESS, K3/S**3	-1.692-01
COEFFICIENT OF EXCESS, K4/S**4	5.028-01
SECOND CENTRAL MOMENT, C2	5.177-01
THIRD CENTRAL MOMENT, C3	-6.302-02
FOURTH CENTRAL MOMENT, C4	9.390-01
FOURTH CUMULANT, K4=(C4-3*C2**2)	1.348-01
SECOND MOMENT ABOUT ZERO, M2	1.097+01
THIRD MOMENT ABOUT ZERO, M3	3.875+01
FOURTH MOMENT ABOUT ZERO, M4	1.418+02
MINIMUM VALUE	6.931-01
MAXIMUM VALUE	5.053+00

LN(RMS) FOR BENDING STRESS

FULL LOAD CONDITIONS

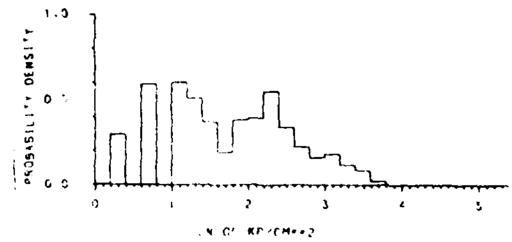


STATISTICAL PARAMETERS

NUMBER OF SAMPLE VALUES	473
ARITHMETIC MEAN VALUE, M1	3.159+00
STANDARD DEVIATION, S	7.602-01
COEFFICIENT OF VARIATION, M1/S	4.155+00
COEFFICIENT OF SKEWNESS, K3/S**3	-2.969-01
COEFFICIENT OF EXCESS, K4/S**4	8.091-01
SECOND CENTRAL MOMENT, C2	5.780-01
THIRD CENTRAL MOMENT, C3	-1.305-01
FOURTH CENTRAL MOMENT, C4	1.272+00
FOURTH CUMULANT, K4=(C4-3*C2**2)	2.703-01
SECOND MOMENT ABOUT ZERO, M2	1.056+01
THIRD MOMENT ABOUT ZERO, M3	3.484+01
FOURTH MOMENT ABOUT ZERO, M4	1.337+02
MINIMUM VALUE	[illegible]
MAXIMUM VALUE	5.033+00

LN(RMS) FOR SPRINGING STRESS

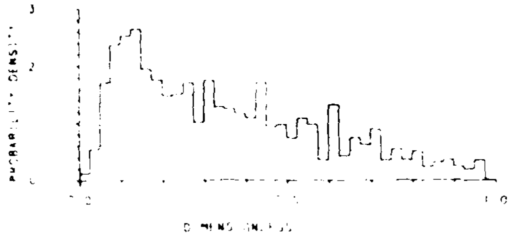
FULL LOAD CONDITIONS



STATISTICAL PARAMETERS

NUMBER OF SAMPLE VALUES	473
ARITHMETIC MEAN VALUE, M1	1.746+00
STANDARD DEVIATION, S	8.246-01
COEFFICIENT OF VARIATION, M1/S	2.117+00
COEFFICIENT OF SKEWNESS, K3/S**3	1.909-01
COEFFICIENT OF EXCESS, K4/S**4	-8.382-01
SECOND CENTRAL MOMENT, C2	6.799-01
THIRD CENTRAL MOMENT, C3	1.070-01
FOURTH CENTRAL MOMENT, C4	9.994-01
FOURTH CUMULANT, K4=(C4-3*C2**2)	-3.875-01
SECOND MOMENT ABOUT ZERO, M2	3.727+00
THIRD MOMENT ABOUT ZERO, M3	8.981+00
FOURTH MOMENT ABOUT ZERO, M4	2.343+01
MINIMUM VALUE	3.466-01
MAXIMUM VALUE	3.748+00

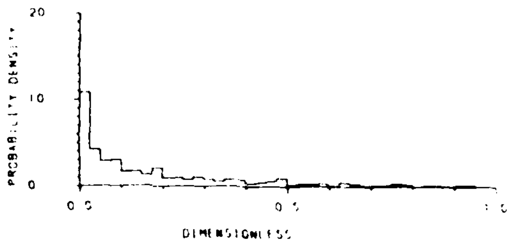
SPRINGING SHARE, X
ALL LOADING CONDITIONS



STATISTICAL PARAMETERS

NUMBER OF SAMPLE VALUES	1030
ARITHMETIC MEAN VALUE, M1	3.561-01
STANDARD DEVIATION, S	2.340-01
COEFFICIENT OF VARIATION, M1/S	1.522+00
COEFFICIENT OF SKEWNESS, K3/S**3	6.720-01
COEFFICIENT OF EXCESS, K4/S**4	-4.780-01
SECOND CENTRAL MOMENT, C2	5.477-02
THIRD CENTRAL MOMENT, C3	8.619-03
FOURTH CENTRAL MOMENT, C4	7.569-03
FOURTH CUMULANT, K4=(C4-3*C2**2)	-1.436-03
SECOND MOMENT ABOUT ZERO, M2	1.815-01
THIRD MOMENT ABOUT ZERO, M3	1.122-01
FOURTH MOMENT ABOUT ZERO, M4	7.744-02
MINIMUM VALUE	1.282-02
MAXIMUM VALUE	9.744-01

SPRINGING SHARE SQUARED, X**2
ALL LOADING CONDITIONS



STATISTICAL PARAMETERS

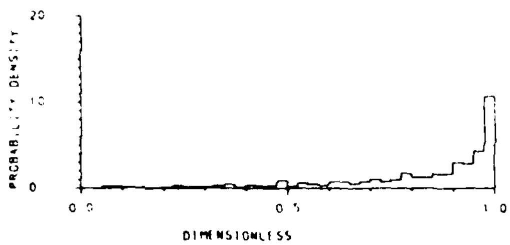
NUMBER OF SAMPLE VALUES	1030
ARITHMETIC MEAN VALUE, M1	1.815-01
STANDARD DEVIATION, S	2.110-01
COEFFICIENT OF VARIATION, M1/S	8.601-01
COEFFICIENT OF SKEWNESS, K3/S**3	1.647+00
COEFFICIENT OF EXCESS, K4/S**4	1.845+00
SECOND CENTRAL MOMENT, C2	4.454-02
THIRD CENTRAL MOMENT, C3	1.454-02
FOURTH CENTRAL MOMENT, C4	9.611-03
FOURTH CUMULANT, K4=(C4-3*C2**2)	3.659-03
SECOND MOMENT ABOUT ZERO, M2	7.744-02
THIRD MOMENT ABOUT ZERO, M3	4.467-02
FOURTH MOMENT ABOUT ZERO, M4	2.996-02
MINIMUM VALUE	1.644-04
MAXIMUM VALUE	9.495-01

BENDING SHARE, XB=SQRT(1-X**2)
ALL LOADING CONDITIONS

STATISTICAL PARAMETERS

NUMBER OF SAMPLE VALUES	1030
ARITHMETIC MEAN VALUE, M1	8.938-01
STANDARD DEVIATION, S	1.402-01
COEFFICIENT OF VARIATION, M1/S	6.377+00
COEFFICIENT OF SKEWNESS, K3/S**3	-2.127+00
COEFFICIENT OF EXCESS, K4/S**4	4.877+00
SECOND CENTRAL MOMENT, C2	1.965-02
THIRD CENTRAL MOMENT, C3	-5.858-03
FOURTH CENTRAL MOMENT, C4	3.041-03
FOURTH CUMULANT, K4=(C4-3*C2**2)	1.883-03
SECOND MOMENT ABOUT ZERO, M2	8.185-01
THIRD MOMENT ABOUT ZERO, M3	7.608-01
FOURTH MOMENT ABOUT ZERO, M4	7.144-01
MINIMUM VALUE	2.249-01
MAXIMUM VALUE	9.999-01

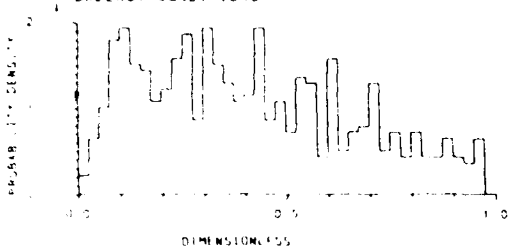
BENDING SHARE SQUARED, XB**2=(1-X**2)
ALL LOADING CONDITIONS



STATISTICAL PARAMETERS

NUMBER OF SAMPLE VALUES	1030
ARITHMETIC MEAN VALUE, M1	8.185-01
STANDARD DEVIATION, S	2.110-01
COEFFICIENT OF VARIATION, M1/S	3.878+00
COEFFICIENT OF SKEWNESS, K3/S**3	-1.547+00
COEFFICIENT OF EXCESS, K4/S**4	1.845+00
SECOND CENTRAL MOMENT, C2	4.454-02
THIRD CENTRAL MOMENT, C3	-1.4[illegible]-02
FOURTH CENTRAL MOMENT, C4	9.613-03
FOURTH CUMULANT, K4=(C4-3*C2**2)	[illegible]
SECOND MOMENT ABOUT ZERO, M2	[illegible]-01
THIRD MOMENT ABOUT ZERO, M3	[illegible]-01
FOURTH MOMENT ABOUT ZERO, M4	[illegible]-01
MINIMUM VALUE	[illegible]-02
MAXIMUM VALUE	9.990-01

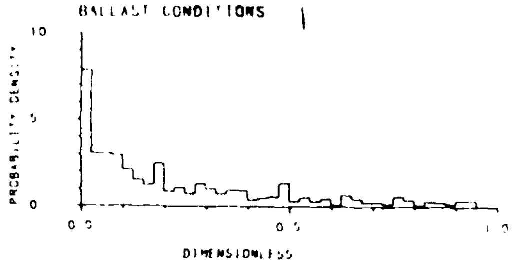
SPRINGING SHARE, X
BALLAST CONDITIONS



STATISTICAL PARAMETERS

NUMBER OF SAMPLE VALUES	557
ARITHMETIC MEAN VALUE, M1	4.139-01
STANDARD DEVIATION, S	2.486-01
COEFFICIENT OF VARIATION, M1/S	1.665+00
COEFFICIENT OF SKEWNESS, K3/S**3	4.180-01
COEFFICIENT OF EXCESS, K4/S**4	-7.973-01
SECOND CENTRAL MOMENT, C2	6.179-02
THIRD CENTRAL MOMENT, C3	6.433-03
FOURTH CENTRAL MOMENT, C4	8.411-03
FOURTH CUMULANT, K4=(C4-3*C2**2)	-3.044-03
SECOND MOMENT ABOUT ZERO, M2	2.330-01
THIRD MOMENT ABOUT ZERO, M3	1.539-01
FOURTH MOMENT ABOUT ZERO, M4	1.116-01
MINIMUM VALUE	1.282-02
MAXIMUM VALUE	9.744-01

SPRINGING SHARE SQUARED, X**2
BALLAST CONDITIONS



STATISTICAL PARAMETERS

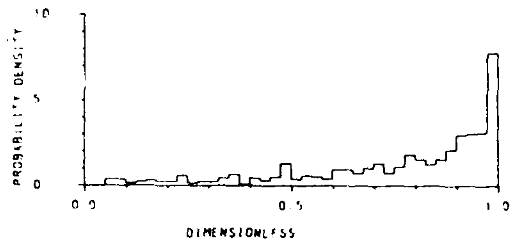
NUMBER OF SAMPLE VALUES	557
ARITHMETIC MEAN VALUE, M1	2.330-01
STANDARD DEVIATION, S	2.397-01
COEFFICIENT OF VARIATION, M1/S	9.721-01
COEFFICIENT OF SKEWNESS, K3/S**3	1.206+00
COEFFICIENT OF EXCESS, K4/S**4	5.950-01
SECOND CENTRAL MOMENT, C2	5.745-02
THIRD CENTRAL MOMENT, C3	1.661-02
FOURTH CENTRAL MOMENT, C4	1.106-02
FOURTH CUMULANT, K4=(C4-3*C2**2)	1.964-03
SECOND MOMENT ABOUT ZERO, M2	1.116-01
THIRD MOMENT ABOUT ZERO, M3	6.922-02
FOURTH MOMENT ABOUT ZERO, M4	4.873-02
MINIMUM VALUE	1.644-04
MAXIMUM VALUE	9.495-01

BENDING SHARE, XB=SQRT(1-X**2)
BALLAST CONDITIONS

STATISTICAL PARAMETERS

NUMBER OF SAMPLE VALUES	557
ARITHMETIC MEAN VALUE, M1	8.602-01
STANDARD DEVIATION, S	1.649-01
COEFFICIENT OF VARIATION, M1/S	5.217+00
COEFFICIENT OF SKEWNESS, K3/S**3	-1.721+00
COEFFICIENT OF EXCESS, K4/S**4	2.640+00
SECOND CENTRAL MOMENT, C2	2.718-02
THIRD CENTRAL MOMENT, C3	-7.712-03
FOURTH CENTRAL MOMENT, C4	4.166-03
FOURTH CUMULANT, K4=(C4-3*C2**2)	1.950-03
SECOND MOMENT ABOUT ZERO, M2	7.670-01
THIRD MOMENT ABOUT ZERO, M3	6.988-01
FOURTH MOMENT ABOUT ZERO, M4	6.456-01
MINIMUM VALUE	2.249-01
MAXIMUM VALUE	9.999-01

BENDING SHARE SQUARED, XB**2=(1-X**2)
BALLAST CONDITIONS



STATISTICAL PARAMETERS

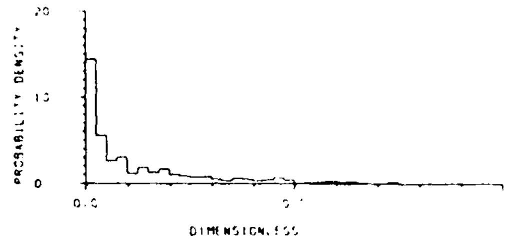
NUMBER OF SAMPLE VALUES	557
ARITHMETIC MEAN VALUE, M1	7.670-01
STANDARD DEVIATION, S	2.397-01
COEFFICIENT OF VARIATION, M1/S	3.200+00
COEFFICIENT OF SKEWNESS, K3/S**3	-1.206+00
COEFFICIENT OF EXCESS, K4/S**4	5.950-01
SECOND CENTRAL MOMENT, C2	5.745-02
THIRD CENTRAL MOMENT, C3	-1.661-02
FOURTH CENTRAL MOMENT, C4	1.106-02
FOURTH CUMULANT, K4=(C4-3*C2**2)	1.964-03
SECOND MOMENT ABOUT ZERO, M2	6.456-01
THIRD MOMENT ABOUT ZERO, M3	5.467-01
FOURTH MOMENT ABOUT ZERO, M4	5.097-01
MINIMUM VALUE	5.050-02
MAXIMUM VALUE	9.998-01

SPRINGING SHARE, X
FULL LOAD CONDITIONS

STATISTICAL PARAMETERS

NUMBER OF SAMPLE VALUES	473
ARITHMETIC MEAN VALUE, M1	2.880-01
STANDARD DEVIATION, S	1.950-01
COEFFICIENT OF VARIATION, M1/S	1.477+00
COEFFICIENT OF SKEWNESS, K3/S**3	8.711-01
COEFFICIENT OF EXCESS, K4/S**4	-1.683-01
SECOND CENTRAL MOMENT, C2	3.803-02
THIRD CENTRAL MOMENT, C3	6.461-03
FOURTH CENTRAL MOMENT, C4	4.096-03
FOURTH CUMULANT, K4=(C4-3*C2**2)	-2.434-04
SECOND MOMENT ABOUT ZERO, M2	1.209-01
THIRD MOMENT ABOUT ZERO, M3	6.307-02
FOURTH MOMENT ABOUT ZERO, M4	3.717-02
MINIMUM VALUE	4.051-02
MAXIMUM VALUE	6.573-01

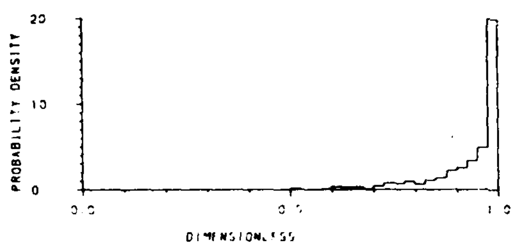
SPRINGING SHARE SQUARED, X**2
FULL LOAD CONDITIONS



STATISTICAL PARAMETERS

NUMBER OF SAMPLE VALUES	473
ARITHMETIC MEAN VALUE, M1	1.209-01
STANDARD DEVIATION, S	1.504-01
COEFFICIENT OF VARIATION, M1/S	8.038-01
COEFFICIENT OF SKEWNESS, K3/S**3	1.737+00
COEFFICIENT OF EXCESS, K4/S**4	2.577+00
SECOND CENTRAL MOMENT, C2	2.262-02
THIRD CENTRAL MOMENT, C3	5.911-03
FOURTH CENTRAL MOMENT, C4	2.054-03
FOURTH CUMULANT, K4=(C4-3*C2**2)	1.319-03
SECOND MOMENT ABOUT ZERO, M2	3.719-02
THIRD MOMENT ABOUT ZERO, M3	1.582-02
FOURTH MOMENT ABOUT ZERO, M4	7.081-03
MINIMUM VALUE	1.641-03
MAXIMUM VALUE	7.353-01

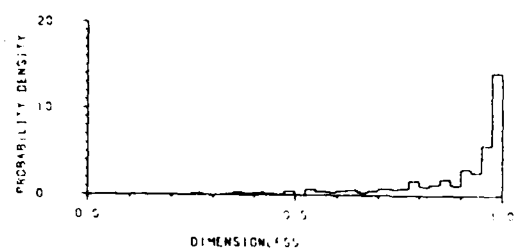
BENDING SHARE, XB=SQRT(1-X**2)
FULL LOAD CONDITIONS



STATISTICAL PARAMETERS

NUMBER OF SAMPLE VALUES	473
ARITHMETIC MEAN VALUE, M1	9.334-01
STANDARD DEVIATION, S	8.893-02
COEFFICIENT OF VARIATION, M1/S	1.050+01
COEFFICIENT OF SKEWNESS, K3/S**3	-2.339+00
COEFFICIENT OF EXCESS, K4/S**4	4.200+00
SECOND CENTRAL MOMENT, C2	7.908-03
THIRD CENTRAL MOMENT, C3	-1.434-03
FOURTH CENTRAL MOMENT, C4	4.502-04
FOURTH CUMULANT, K4=(C4-3*C2**2)	2.626-04
SECOND MOMENT ABOUT ZERO, M2	8.791-01
THIRD MOMENT ABOUT ZERO, M3	8.334-01
FOURTH MOMENT ABOUT ZERO, M4	7.954-01
MINIMUM VALUE	5.195-01
MAXIMUM VALUE	9.992-01

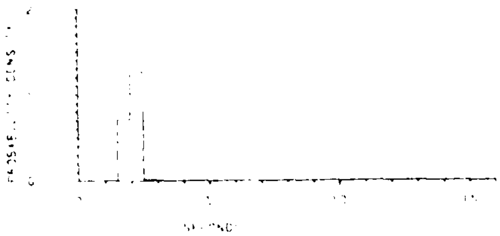
BENDING SHARE SQUARED, XB**2=(1-X**2)
FULL LOAD CONDITIONS



STATISTICAL PARAMETERS

NUMBER OF SAMPLE VALUES	473
ARITHMETIC MEAN VALUE, M1	8.791-01
STANDARD DEVIATION, S	1.504-01
COEFFICIENT OF VARIATION, M1/S	5.845+00
COEFFICIENT OF SKEWNESS, K3/S**3	-1.737+00
COEFFICIENT OF EXCESS, K4/S**4	2.579+00
SECOND CENTRAL MOMENT, C2	2.262-02
THIRD CENTRAL MOMENT, C3	-5.911-03
FOURTH CENTRAL MOMENT, C4	2.485-03
FOURTH CUMULANT, K4=(C4-3*C2**2)	1.320-03
SECOND MOMENT ABOUT ZERO, M2	7.954-01
THIRD MOMENT ABOUT ZERO, M3	7.330-01
FOURTH MOMENT ABOUT ZERO, M4	6.841-01
MINIMUM VALUE	2.647-01
MAXIMUM VALUE	9.984-01

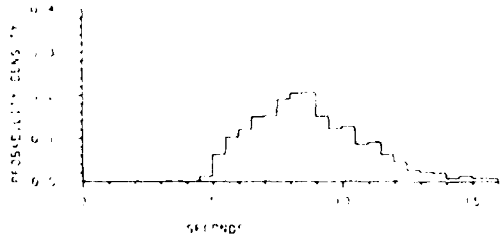
AV SPRINGING PERIOD, TS
ALL LOADING CONDITIONS



STATISTICAL PARAMETERS

NUMBER OF SAMPLE VALUES	1024
ARITHMETIC MEAN VALUE, M1	2.115+00
STANDARD DEVIATION, S	2.357-01
COEFFICIENT OF VARIATION, M1/S	8.973+00
COEFFICIENT OF SKEWNESS, K3/S**3	1.412-02
COEFFICIENT OF EXCESS, K4/S**4	-1.549+00
SECOND CENTRAL MOMENT, C2	5.554-02
THIRD CENTRAL MOMENT, C3	1.844-04
FOURTH CENTRAL MOMENT, C4	4.478-03
FOURTH CUMULANT, K4=(C4-3*C2**2)	-4.775-03
SECOND MOMENT ABOUT ZERO, M2	4.529+00
THIRD MOMENT ABOUT ZERO, M3	9.810+00
FOURTH MOMENT ABOUT ZERO, M4	2.143+01
MINIMUM VALUE	1.700+00
MAXIMUM VALUE	2.800+00

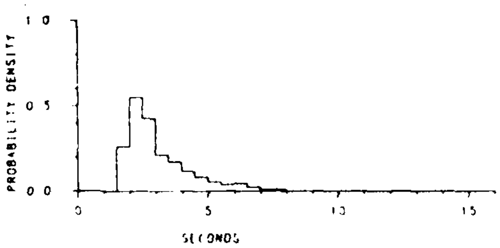
AV BENDING PERIOD, TB
ALL LOADING CONDITIONS



STATISTICAL PARAMETERS

NUMBER OF SAMPLE VALUES	1024
ARITHMETIC MEAN VALUE, M1	[illegible]
STANDARD DEVIATION, S	2.120+00
COEFFICIENT OF VARIATION, M1/S	4.085+00
COEFFICIENT OF SKEWNESS, K3/S**3	6.219-01
COEFFICIENT OF EXCESS, K4/S**4	3.021-01
SECOND CENTRAL MOMENT, C2	4.530+00
THIRD CENTRAL MOMENT, C3	[illegible]
FOURTH CENTRAL MOMENT, C4	6.776+01
FOURTH CUMULANT, K4=(C4-3*C2**2)	6.198+00
SECOND MOMENT ABOUT ZERO, M2	8.013+01
THIRD MOMENT ABOUT ZERO, M3	[illegible]
FOURTH MOMENT ABOUT ZERO, M4	[illegible]
MINIMUM VALUE	[illegible]
MAXIMUM VALUE	[illegible]

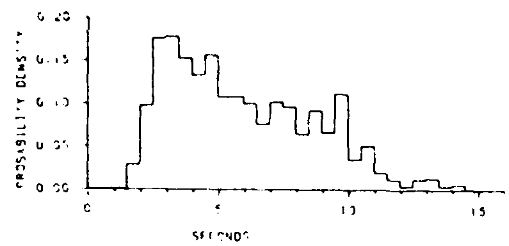
AV.PEAK PERIOD, TP
ALL LOADING CONDITIONS



STATISTICAL PARAMETERS

NUMBER OF SAMPLE VALUES	1030
ARITHMETIC MEAN VALUE, M1	3.095+00
STANDARD DEVIATION, S	1.256+00
COEFFICIENT OF VARIATION, M1/S	2.465+00
COEFFICIENT OF SKEWNESS, K3/S**3	1.796+00
COEFFICIENT OF EXCESS, K4/S**4	4.689+00
SECOND CENTRAL MOMENT, C2	1.577+00
THIRD CENTRAL MOMENT, C3	3.556+00
FOURTH CENTRAL MOMENT, C4	1.912+01
FOURTH CUMULANT, K4=(C4-3*C2**2)	1.165+01
SECOND MOMENT ABOUT ZERO, M2	1.116+01
THIRD MOMENT ABOUT ZERO, M3	4.782+01
FOURTH MOMENT ABOUT ZERO, M4	2.452+02
MINIMUM VALUE	1.765+00
MAXIMUM VALUE	1.229+01

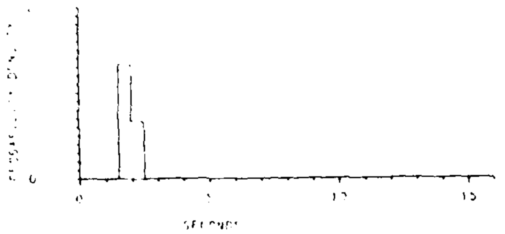
AV.ZERO CROSSING PERIOD, TZ
ALL LOADING CONDITIONS



STATISTICAL PARAMETERS

NUMBER OF SAMPLE VALUES	1024
ARITHMETIC MEAN VALUE, M1	5.911+00
STANDARD DEVIATION, S	2.749+00
COEFFICIENT OF VARIATION, M1/S	2.151+00
COEFFICIENT OF SKEWNESS, K3/S**3	5.681-01
COEFFICIENT OF EXCESS, K4/S**4	-5.170-01
SECOND CENTRAL MOMENT, C2	7.556+00
THIRD CENTRAL MOMENT, C3	1.180+01
FOURTH CENTRAL MOMENT, C4	1.417+02
FOURTH CUMULANT, K4=(C4-3*C2**2)	-2.954+01
SECOND MOMENT ABOUT ZERO, M2	4.249+01
THIRD MOMENT ABOUT ZERO, M3	3.522+02
FOURTH MOMENT ABOUT ZERO, M4	3.222+03
MINIMUM VALUE	1.083+00
MAXIMUM VALUE	1.424+01

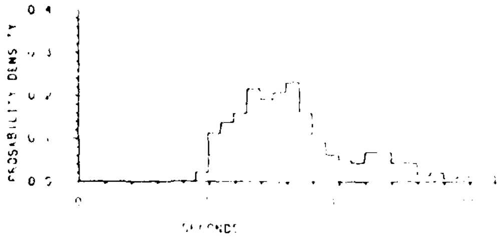
AV SPRINGING PERIOD TS
BALLAST CONDITIONS



STATISTICAL PARAMETERS

NUMBER OF SAMPLE VALUES	551
ARITHMETIC MEAN VALUE, M1	1.912+00
STANDARD DEVIATION, S	1.030-01
COEFFICIENT OF VARIATION, M1/S	1.857+01
COEFFICIENT OF SKEWNESS, K3/S**3	8.363-01
COEFFICIENT OF EXCESS, K4/S**4	1.486+00
SECOND CENTRAL MOMENT, C2	1.060-02
THIRD CENTRAL MOMENT, C3	9.127-04
FOURTH CENTRAL MOMENT, C4	5.041-04
FOURTH CUMULANT, K4=(C4-3*C2**2)	1.670-04
SECOND MOMENT ABOUT ZERO, M2	3.667+00
THIRD MOMENT ABOUT ZERO, M3	7.053+00
FOURTH MOMENT ABOUT ZERO, M4	1.361+01
MINIMUM VALUE	1.700+00
MAXIMUM VALUE	2.400+00

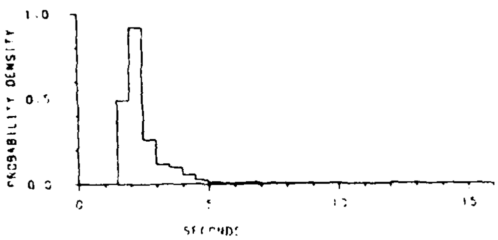
AV BENDING PERIOD TB
BALLAST CONDITIONS



STATISTICAL PARAMETERS

NUMBER OF SAMPLE VALUES	551
ARITHMETIC MEAN VALUE, M1	8.264+00
STANDARD DEVIATION, S	2.295+00
COEFFICIENT OF VARIATION, M1/S	3.600+00
COEFFICIENT OF SKEWNESS, K3/S**3	1.016+00
COEFFICIENT OF EXCESS, K4/S**4	7.747-01
SECOND CENTRAL MOMENT, C2	5.268+00
THIRD CENTRAL MOMENT, C3	1.227+01
FOURTH CENTRAL MOMENT, C4	1.049+02
FOURTH CUMULANT, K4=(C4-3*C2**2)	2.150+01
SECOND MOMENT ABOUT ZERO, M2	7.355+01
THIRD MOMENT ABOUT ZERO, M3	7.067+02
FOURTH MOMENT ABOUT ZERO, M4	7.325+03
MINIMUM VALUE	4.600+00
MAXIMUM VALUE	1.620+01

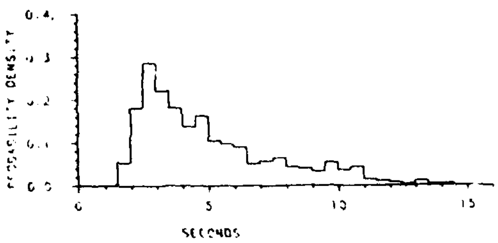
AV PEAK PERIOD TP
BALLAST CONDITIONS



STATISTICAL PARAMETERS

NUMBER OF SAMPLE VALUES	551
ARITHMETIC MEAN VALUE, M1	2.496+00
STANDARD DEVIATION, S	8.044-01
COEFFICIENT OF VARIATION, M1/S	3.102+00
COEFFICIENT OF SKEWNESS, K3/S**3	2.524+00
COEFFICIENT OF EXCESS, K4/S**4	8.020+00
SECOND CENTRAL MOMENT, C2	6.471-01
THIRD CENTRAL MOMENT, C3	1.313+00
FOURTH CENTRAL MOMENT, C4	4.614+00
FOURTH CUMULANT, K4=(C4-3*C2**2)	3.358+00
SECOND MOMENT ABOUT ZERO, M2	6.874+00
THIRD MOMENT ABOUT ZERO, M3	2.163+01
FOURTH MOMENT ABOUT ZERO, M4	9.052+01
MINIMUM VALUE	1.765+00
MAXIMUM VALUE	7.234+00

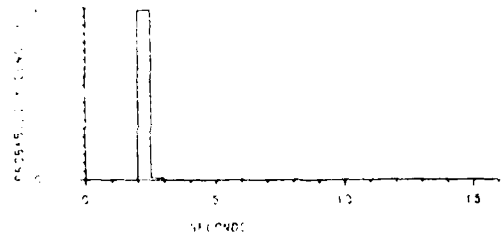
AV ZERO CROSSING PERIOD TZ
BALLAST CONDITIONS



STATISTICAL PARAMETERS

NUMBER OF SAMPLE VALUES	551
ARITHMETIC MEAN VALUE, M1	5.034+00
STANDARD DEVIATION, S	2.640+00
COEFFICIENT OF VARIATION, M1/S	1.907+00
COEFFICIENT OF SKEWNESS, K3/S**3	1.070+00
COEFFICIENT OF EXCESS, K4/S**4	4.402-01
SECOND CENTRAL MOMENT, C2	6.971+00
THIRD CENTRAL MOMENT, C3	1.968+01
FOURTH CENTRAL MOMENT, C4	1.672+02
FOURTH CUMULANT, K4=(C4-3*C2**2)	2.134+01
SECOND MOMENT ABOUT ZERO, M2	3.230+01
THIRD MOMENT ABOUT ZERO, M3	2.522+02
FOURTH MOMENT ABOUT ZERO, M4	2.259+03
MINIMUM VALUE	1.851+00
MAXIMUM VALUE	1.412+01

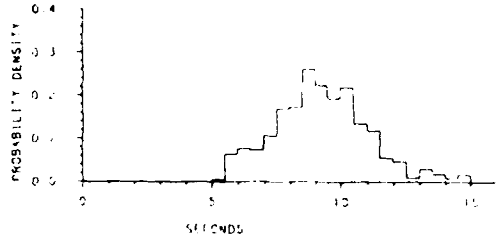
AV SPRINGING PERIOD, TS
FULL LOAD CONDITIONS



STATISTICAL PARAMETERS

NUMBER OF SAMPLE VALUES	473
ARITHMETIC MEAN VALUE, M1	2.351+00
STANDARD DEVIATION, S	6.552-02
COEFFICIENT OF VARIATION, M1/S	3.588+01
COEFFICIENT OF SKEWNESS, K3/S**3	1.192+00
COEFFICIENT OF EXCESS, K4/S**4	5.939+00
SECOND CENTRAL MOMENT, C2	4.292-03
THIRD CENTRAL MOMENT, C3	3.371-04
FOURTH CENTRAL MOMENT, C4	1.647-04
FOURTH CUMULANT, K4=(C4-3*C2**2)	1.094-04
SECOND MOMENT ABOUT ZERO, M2	5.530+00
THIRD MOMENT ABOUT ZERO, M3	1.302+01
FOURTH MOMENT ABOUT ZERO, M4	3.068+01
MINIMUM VALUE	2.200+00
MAXIMUM VALUE	2.800+00

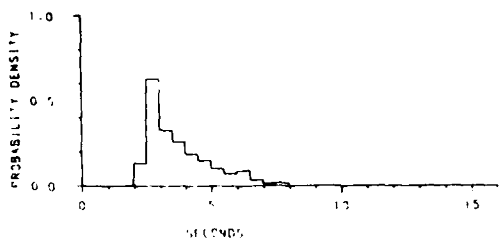
AV BENDING PERIOD, TB
FULL LOAD CONDITIONS



STATISTICAL PARAMETERS

NUMBER OF SAMPLE VALUES	473
ARITHMETIC MEAN VALUE, M1	9.196+00
STANDARD DEVIATION, S	1.791+00
COEFFICIENT OF VARIATION, M1/S	5.135+00
COEFFICIENT OF SKEWNESS, K3/S**3	3.142-01
COEFFICIENT OF EXCESS, K4/S**4	3.612-01
SECOND CENTRAL MOMENT, C2	3.204+00
THIRD CENTRAL MOMENT, C3	1.806+00
FOURTH CENTRAL MOMENT, C4	3.460+01
FOURTH CUMULANT, K4=(C4-3*C2**2)	3.718+00
SECOND MOMENT ABOUT ZERO, M2	8.790+01
THIRD MOMENT ABOUT ZERO, M3	8.683+02
FOURTH MOMENT ABOUT ZERO, M4	8.803+03
MINIMUM VALUE	5.000+00
MAXIMUM VALUE	1.490+01

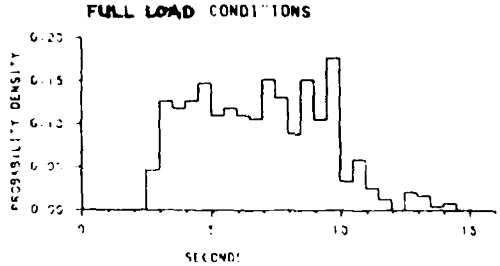
AV.PEAK PERIOD, TP
FULL LOAD CONDITIONS



STATISTICAL PARAMETERS

NUMBER OF SAMPLE VALUES	473
ARITHMETIC MEAN VALUE, M1	3.731+00
STANDARD DEVIATION, S	1.216+00
COEFFICIENT OF VARIATION, M1/S	3.069+00
COEFFICIENT OF SKEWNESS, K3/S**3	1.131+00
COEFFICIENT OF EXCESS, K4/S**4	6.169-01
SECOND CENTRAL MOMENT, C2	1.479+00
THIRD CENTRAL MOMENT, C3	2.033+00
FOURTH CENTRAL MOMENT, C4	7.902+00
FOURTH CUMULANT, K4=(C4-3*C2**2)	1.349+00
SECOND MOMENT ABOUT ZERO, M2	1.540+01
THIRD MOMENT ABOUT ZERO, M3	7.044+01
FOURTH MOMENT ABOUT ZERO, M4	3.353+02
MINIMUM VALUE	2.352+00
MAXIMUM VALUE	7.985+00

AV.ZERO CROSSING PERIOD, TZ
FULL LOAD CONDITIONS



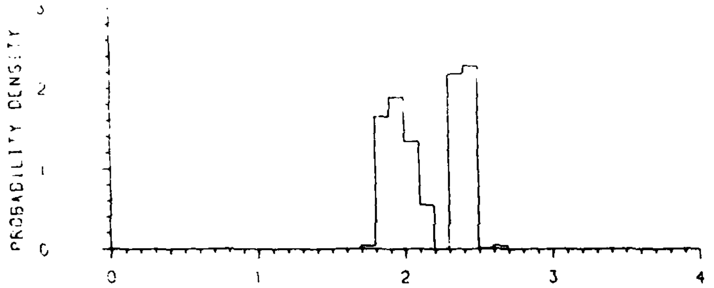
STATISTICAL PARAMETERS

NUMBER OF SAMPLE VALUES	473
ARITHMETIC MEAN VALUE, M1	6.934+00
STANDARD DEVIATION, S	2.511+00
COEFFICIENT OF VARIATION, M1/S	2.761+00
COEFFICIENT OF SKEWNESS, K3/S**3	2.824-01
COEFFICIENT OF EXCESS, K4/S**4	-5.717-01
SECOND CENTRAL MOMENT, C2	6.307+00
THIRD CENTRAL MOMENT, C3	4.473+00
FOURTH CENTRAL MOMENT, C4	9.658+01
FOURTH CUMULANT, K4=(C4-3*C2**2)	-2.274+01
SECOND MOMENT ABOUT ZERO, M2	5.437+01
THIRD MOMENT ABOUT ZERO, M3	4.687+02
FOURTH MOMENT ABOUT ZERO, M4	4.345+03
MINIMUM VALUE	2.612+00
MAXIMUM VALUE	1.424+01

EXPANDED DRAWINGS OF SPRINGING PERIOD

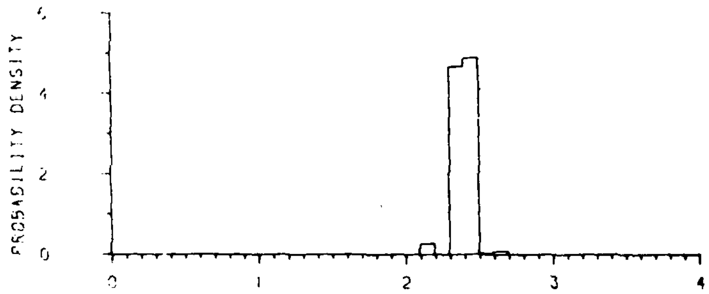
AV. SPRINGING PERIOD, TS

ALL LOADING CONDITIONS



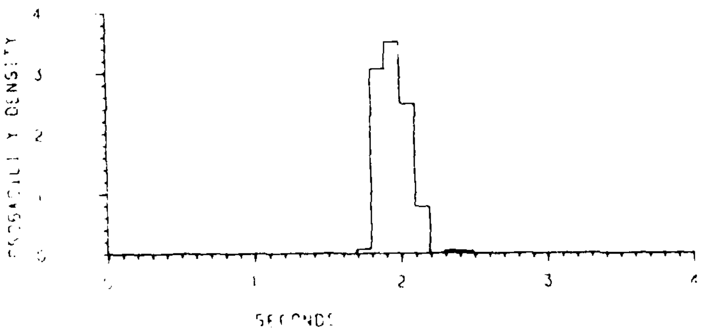
AV. SPRINGING PERIOD, TS

FULL LOAD CONDITIONS



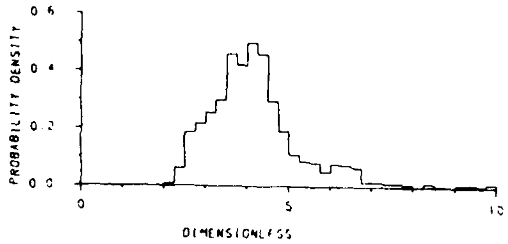
AV. SPRINGING PERIOD, TS

BALLAST CONDITIONS



DIMENSIONLESS BENDING PERIOD. TB/TS TAU

ALL LOADING CONDITIONS

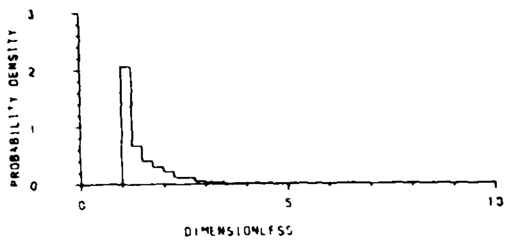


STATISTICAL PARAMETERS

NUMBER OF SAMPLE VALUES	1030
ARITHMETIC MEAN VALUE, M1	4.176+00
STANDARD DEVIATION, S	1.153+00
COEFFICIENT OF VARIATION, M1/S	3.622+00
COEFFICIENT OF SKEWNESS, K3/S**3	1.566+00
COEFFICIENT OF EXCESS, K4/S**4	4.409+00
SECOND CENTRAL MOMENT, C2	1.330+00
THIRD CENTRAL MOMENT, C3	2.401+00
FOURTH CENTRAL MOMENT, C4	1.310+01
FOURTH CUMULANT, K4=(C4-3*C2**2)	7.797+00
SECOND MOMENT ABOUT ZERO, M2	1.877+01
THIRD MOMENT ABOUT ZERO, M3	9.189+01
FOURTH MOMENT ABOUT ZERO, M4	4.963+02
MINIMUM VALUE	2.000+00
MAXIMUM VALUE	1.075+01

DIMENSIONLESS PEAK PERIOD. TP/TS

ALL LOADING CONDITIONS

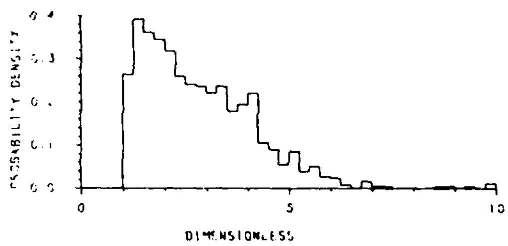


STATISTICAL PARAMETERS

NUMBER OF SAMPLE VALUES	1030
ARITHMETIC MEAN VALUE, M1	1.450+00
STANDARD DEVIATION, S	5.269-01
COEFFICIENT OF VARIATION, M1/S	2.751+00
COEFFICIENT OF SKEWNESS, K3/S**3	2.357+00
COEFFICIENT OF EXCESS, K4/S**4	9.550+00
SECOND CENTRAL MOMENT, C2	2.776-01
THIRD CENTRAL MOMENT, C3	3.448-01
FOURTH CENTRAL MOMENT, C4	9.673-01
FOURTH CUMULANT, K4=(C4-3*C2**2)	7.361-01
SECOND MOMENT ABOUT ZERO, M2	2.379+00
THIRD MOMENT ABOUT ZERO, M3	4.596+00
FOURTH MOMENT ABOUT ZERO, M4	1.087+01
MINIMUM VALUE	1.003+00
MAXIMUM VALUE	6.146+00

DIMENSIONLESS ZERO CROSSING PERIOD. TZ/TS

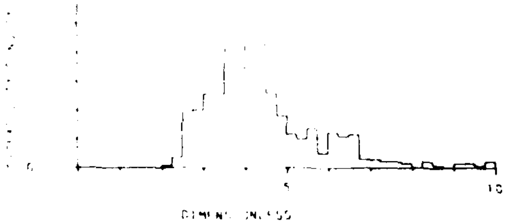
ALL LOADING CONDITIONS



STATISTICAL PARAMETERS

NUMBER OF SAMPLE VALUES	1030
ARITHMETIC MEAN VALUE, M1	2.821+00
STANDARD DEVIATION, S	1.360+00
COEFFICIENT OF VARIATION, M1/S	2.075+00
COEFFICIENT OF SKEWNESS, K3/S**3	1.268+00
COEFFICIENT OF EXCESS, K4/S**4	2.952+00
SECOND CENTRAL MOMENT, C2	1.850+00
THIRD CENTRAL MOMENT, C3	3.190+00
FOURTH CENTRAL MOMENT, C4	2.036+01
FOURTH CUMULANT, K4=(C4-3*C2**2)	1.010+01
SECOND MOMENT ABOUT ZERO, M2	9.808+00
THIRD MOMENT ABOUT ZERO, M3	[illegible]
FOURTH MOMENT ABOUT ZERO, M4	2.078+02
MINIMUM VALUE	[illegible]
MAXIMUM VALUE	[illegible]

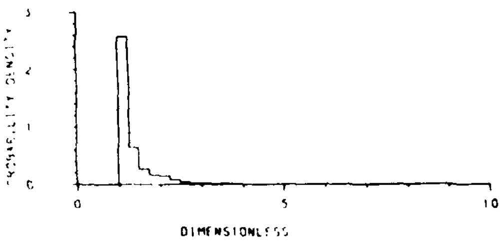
DIMENSIONLESS BENDING PERIOD TB/TS-TAU
BALLAST CONDITIONS



STATISTICAL PARAMETERS

NUMBER OF SAMPLE VALUES	557
ARITHMETIC MEAN VALUE, M1	4.395+00
STANDARD DEVIATION, S	1.352+00
COEFFICIENT OF VARIATION, M1/S	3.252+00
COEFFICIENT OF SKEWNESS, K3/S**3	1.415+00
COEFFICIENT OF EXCESS, K4/S**4	2.856+00
SECOND CENTRAL MOMENT, C2	1.827+00
THIRD CENTRAL MOMENT, C3	3.494+00
FOURTH CENTRAL MOMENT, C4	1.954+01
FOURTH CUMULANT, K4=(C4-3*C2**2)	[illegible].531+00
SECOND MOMENT ABOUT ZERO, M2	2.110+01
THIRD MOMENT ABOUT ZERO, M3	[illegible]
FOURTH MOMENT ABOUT ZERO, M4	6.699+02
MINIMUM VALUE	[illegible]
MAXIMUM VALUE	[illegible]

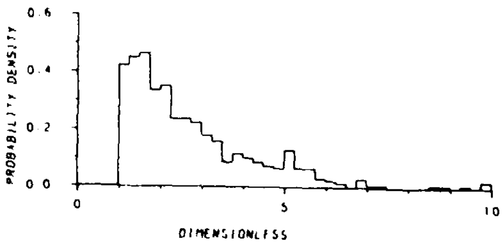
DIMENSIONLESS PEAK PERIOD, TP/TS
BALLAST CONDITIONS



STATISTICAL PARAMETERS

NUMBER OF SAMPLE VALUES	557
ARITHMETIC MEAN VALUE, M1	1.331+00
STANDARD DEVIATION, S	4.997-01
COEFFICIENT OF VARIATION, M1/S	2.663+00
COEFFICIENT OF SKEWNESS, K3/S**3	3.924+00
COEFFICIENT OF EXCESS, K4/S**4	2.367+01
SECOND CENTRAL MOMENT, C2	2.499-01
THIRD CENTRAL MOMENT, C3	4.902-01
FOURTH CENTRAL MOMENT, C4	1.665+00
FOURTH CUMULANT, K4=(C4-3*C2**2)	1.478+00
SECOND MOMENT ABOUT ZERO, M2	2.021+00
THIRD MOMENT ABOUT ZERO, M3	3.840+00
FOURTH MOMENT ABOUT ZERO, M4	1.003+01
MINIMUM VALUE	1.003+00
MAXIMUM VALUE	6.146+00

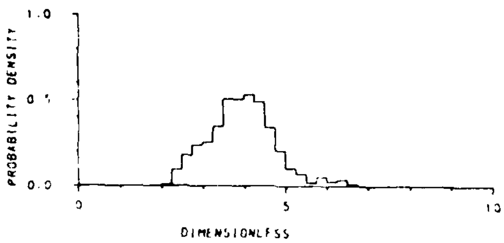
DIMENSIONLESS ZERO CROSSING PERIOD, TZ/TS
BALLAST CONDITIONS



STATISTICAL PARAMETERS

NUMBER OF SAMPLE VALUES	557
ARITHMETIC MEAN VALUE, M1	2.787+00
STANDARD DEVIATION, S	1.544+00
COEFFICIENT OF VARIATION, M1/S	1.753+00
COEFFICIENT OF SKEWNESS, K3/S**3	1.610+00
COEFFICIENT OF EXCESS, K4/S**4	3.505+00
SECOND CENTRAL MOMENT, C2	2.385+00
THIRD CENTRAL MOMENT, C3	5.960+00
FOURTH CENTRAL MOMENT, C4	3.700+01
FOURTH CUMULANT, K4=(C4-3*C2**2)	1.994+01
SECOND MOMENT ABOUT ZERO, M2	9.767+00
THIRD MOMENT ABOUT ZERO, M3	4.508+01
FOURTH MOMENT ABOUT ZERO, M4	2.591+02
MINIMUM VALUE	1.023+00
MAXIMUM VALUE	1.032+01

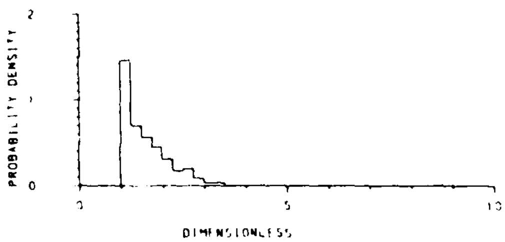
DIMENSIONLESS BENDING PERIOD. TB/TS-TAU
FULL LOAD CONDITIONS



STATISTICAL PARAMETERS

NUMBER OF SAMPLE VALUES	473
ARITHMETIC MEAN VALUE, M1	3.911+00
STANDARD DEVIATION, S	7.900-01
COEFFICIENT OF VARIATION, M1/S	4.961+00
COEFFICIENT OF SKEWNESS, K3/S**3	4.894-01
COEFFICIENT OF EXCESS, K4/S**4	5.096-01
SECOND CENTRAL MOMENT, C2	6.240-01
THIRD CENTRAL MOMENT, C3	2.019-01
FOURTH CENTRAL MOMENT, C4	1.366+00
FOURTH CUMULANT, K4=(C4-3*C2**2)	1.981-01
SECOND MOMENT ABOUT ZERO, M2	1.598+01
THIRD MOMENT ABOUT ZERO, M3	6.770+01
FOURTH MOMENT ABOUT ZERO, M4	2.977+02
MINIMUM VALUE	2.000+00
MAXIMUM VALUE	6.500+00

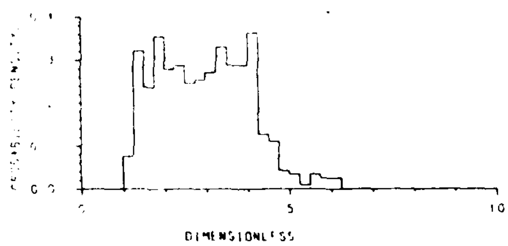
DIMENSIONLESS PEAK PERIOD. TP/TS
FULL LOAD CONDITIONS



STATISTICAL PARAMETERS

NUMBER OF SAMPLE VALUES	473
ARITHMETIC MEAN VALUE, M1	1.598+00
STANDARD DEVIATION, S	5.241-01
COEFFICIENT OF VARIATION, M1/S	3.033+00
COEFFICIENT OF SKEWNESS, K3/S**3	1.147+00
COEFFICIENT OF EXCESS, K4/S**4	6.944-01
SECOND CENTRAL MOMENT, C2	2.747-01
THIRD CENTRAL MOMENT, C3	1.651-01
FOURTH CENTRAL MOMENT, C4	2.780-01
FOURTH CUMULANT, K4=(C4-3*C2**2)	5.163-02
SECOND MOMENT ABOUT ZERO, M2	2.401+00
THIRD MOMENT ABOUT ZERO, M3	5.487+00
FOURTH MOMENT ABOUT ZERO, M4	1.185+01
MINIMUM VALUE	1.023+00
MAXIMUM VALUE	3.413+00

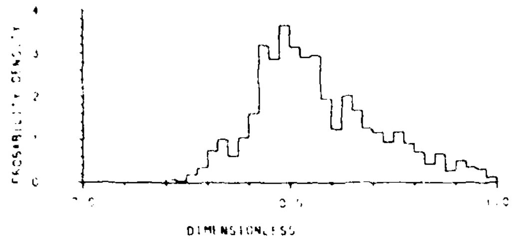
DIMENSIONLESS ZERO CROSSING PERIOD. TZ/TS
FULL LOAD CONDITIONS



STATISTICAL PARAMETERS

NUMBER OF SAMPLE VALUES	473
ARITHMETIC MEAN VALUE, M1	2.957+00
STANDARD DEVIATION, S	1.090+00
COEFFICIENT OF VARIATION, M1/S	2.711+00
COEFFICIENT OF SKEWNESS, K3/S**3	3.381-01
COEFFICIENT OF EXCESS, K4/S**4	-4.569-01
SECOND CENTRAL MOMENT, C2	1.189+00
THIRD CENTRAL MOMENT, C3	4.383-01
FOURTH CENTRAL MOMENT, C4	3.595+00
FOURTH CUMULANT, K4=(C4-3*C2**2)	-6.460-01
SECOND MOMENT ABOUT ZERO, M2	9.928+00
THIRD MOMENT ABOUT ZERO, M3	3.688+01
FOURTH MOMENT ABOUT ZERO, M4	1.473+02
MINIMUM VALUE	1.135+00
MAXIMUM VALUE	6.192+00

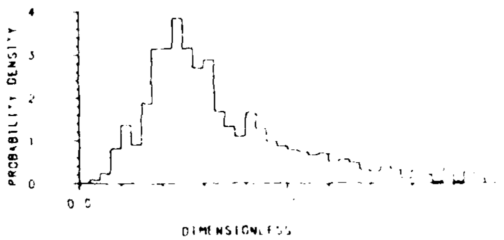
PERIOD RATIO. ALPHA-TP TZ
ALL LOADING CONDITIONS



STATISTICAL PARAMETERS

NUMBER OF SAMPLE VALUES	1030
ARITHMETIC MEAN VALUE, M1	5.642-01
STANDARD DEVIATION, S	1.499-01
COEFFICIENT OF VARIATION, M1/S	3.764+00
COEFFICIENT OF SKEWNESS, K3/S**3	5.753-01
COEFFICIENT OF EXCESS, K4/S**4	-3.232-02
SECOND CENTRAL MOMENT, C2	2.249-02
THIRD CENTRAL MOMENT, C3	1.938-03
FOURTH CENTRAL MOMENT, C4	1.499-03
FOURTH CUMULANT, K4=(C4-3*C2**2)	-1.633-05
SECOND MOMENT ABOUT ZERO, M2	3.408-01
THIRD MOMENT ABOUT ZERO, M3	2.196-01
FOURTH MOMENT ABOUT ZERO, M4	1.501-01
MINIMUM VALUE	2.206-01
MAXIMUM VALUE	9.004-01

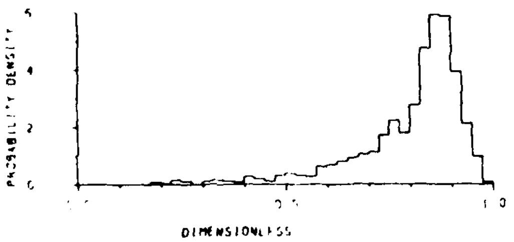
PERIOD RATIO SQUARED ALPHA**2
ALL LOADING CONDITIONS



STATISTICAL PARAMETERS

NUMBER OF SAMPLE VALUES	1030
ARITHMETIC MEAN VALUE, M1	3.409-01
STANDARD DEVIATION, S	1.843-01
COEFFICIENT OF VARIATION, M1/S	1.849+00
COEFFICIENT OF SKEWNESS, K3/S**3	1.164+00
COEFFICIENT OF EXCESS, K4/S**4	1.002+00
SECOND CENTRAL MOMENT, C2	3.397-02
THIRD CENTRAL MOMENT, C3	7.289-03
FOURTH CENTRAL MOMENT, C4	4.619-03
FOURTH CUMULANT, K4=(C4-3*C2**2)	1.157-03
SECOND MOMENT ABOUT ZERO, M2	1.501-01
THIRD MOMENT ABOUT ZERO, M3	4.159-02
FOURTH MOMENT ABOUT ZERO, M4	5.163-02
MINIMUM VALUE	4.866-02
MAXIMUM VALUE	9.612-01

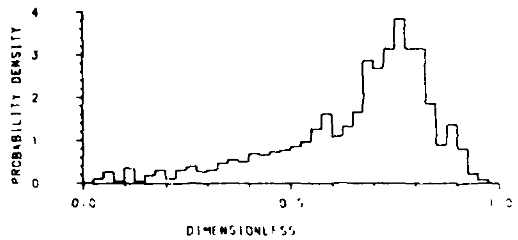
SPECTRAL WIDTH. EPSILON=SQRT(1-(TP/TZ)**2)
ALL LOADING CONDITIONS



STATISTICAL PARAMETERS

NUMBER OF SAMPLE VALUES	1030
ARITHMETIC MEAN VALUE, M1	8.000-01
STANDARD DEVIATION, S	1.338-01
COEFFICIENT OF VARIATION, M1/S	5.963+00
COEFFICIENT OF SKEWNESS, K3/S**3	-1.802+00
COEFFICIENT OF EXCESS, K4/S**4	3.685+00
SECOND CENTRAL MOMENT, C2	1.792-02
THIRD CENTRAL MOMENT, C3	-4.821-03
FOURTH CENTRAL MOMENT, C4	2.149-03
FOURTH CUMULANT, K4=(C4-3*C2**2)	1.183-03
SECOND MOMENT ABOUT ZERO, M2	6.592-01
THIRD MOMENT ABOUT ZERO, M3	5.522-01
FOURTH MOMENT ABOUT ZERO, M4	4.685-01
MINIMUM VALUE	1.969-01
MAXIMUM VALUE	9.784-01

SPECTRAL WIDTH SQUARED. EPSILON**2
ALL LOADING CONDITIONS



STATISTICAL PARAMETERS

NUMBER OF SAMPLE VALUES	1030
ARITHMETIC MEAN VALUE, M1	6.592-01
STANDARD DEVIATION, S	1.843-01
COEFFICIENT OF VARIATION, M1/S	3.576+00
COEFFICIENT OF SKEWNESS, K3/S**3	-1.164+00
COEFFICIENT OF EXCESS, K4/S**4	1.003+00
SECOND CENTRAL MOMENT, C2	3.397-02
THIRD CENTRAL MOMENT, C3	-7.291-03
FOURTH CENTRAL MOMENT, C4	4.620-03
FOURTH CUMULANT, K4=(C4-3*C2**2)	1.159-03
SECOND MOMENT ABOUT ZERO, M2	4.689-01
THIRD MOMENT ABOUT ZERO, M3	3.463-01
FOURTH MOMENT ABOUT ZERO, M4	2.627-01
MINIMUM VALUE	3.977-02
MAXIMUM VALUE	9.514-01

PERIOD RATIO, ALPHA

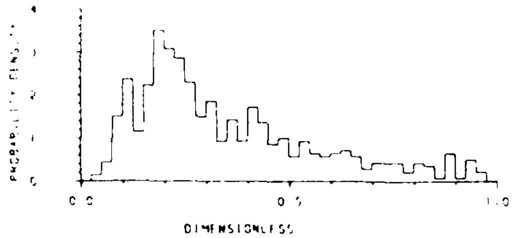
BALLAST CONDITIONS

STATISTICAL PARAMETERS

NUMBER OF SAMPLE VALUES	557
ARITHMETIC MEAN VALUE, M1	5.663-01
STANDARD DEVIATION, S	1.762-01
COEFFICIENT OF VARIATION, M1/S	3.214+00
COEFFICIENT OF SKEWNESS, K3/S**3	4.695-01
COEFFICIENT OF EXCESS, K4/S**4	-5.506-01
SECOND CENTRAL MOMENT, C2	3.104-02
THIRD CENTRAL MOMENT, C3	2.567-03
FOURTH CENTRAL MOMENT, C4	2.357-03
FOURTH CUMULANT, K4=(C4-3*C2**2)	-5.304-04
SECOND MOMENT ABOUT ZERO, M2	3.516-01
THIRD MOMENT ABOUT ZERO, M3	2.368-01
FOURTH MOMENT ABOUT ZERO, M4	1.705-01
MINIMUM VALUE	2.206-01
MAXIMUM VALUE	9.804-01

PERIOD RATIO SQUARED, ALPHA**2

BALLAST CONDITIONS

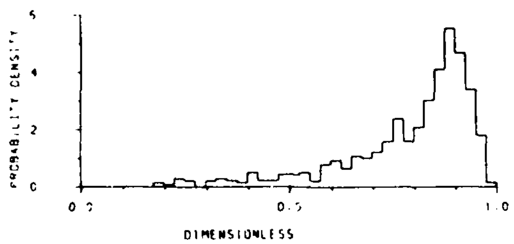


STATISTICAL PARAMETERS

NUMBER OF SAMPLE VALUES	557
ARITHMETIC MEAN VALUE, M1	3.616-01
STANDARD DEVIATION, S	2.167-01
COEFFICIENT OF VARIATION, M1/S	1.623+00
COEFFICIENT OF SKEWNESS, K3/S**3	9.850-01
COEFFICIENT OF EXCESS, K4/S**4	1.987-01
SECOND CENTRAL MOMENT, C2	4.696-02
THIRD CENTRAL MOMENT, C3	1.003-02
FOURTH CENTRAL MOMENT, C4	7.055-03
FOURTH CUMULANT, K4=(C4-3*C2**2)	4.383-04
SECOND MOMENT ABOUT ZERO, M2	1.705-01
THIRD MOMENT ABOUT ZERO, M3	1.029-01
FOURTH MOMENT ABOUT ZERO, M4	7.106-02
MINIMUM VALUE	4.866-02
MAXIMUM VALUE	9.012-01

SPECTRAL WIDTH, EPSILON=SQRT(1-(TP/TZ)**2)

BALLAST CONDITIONS

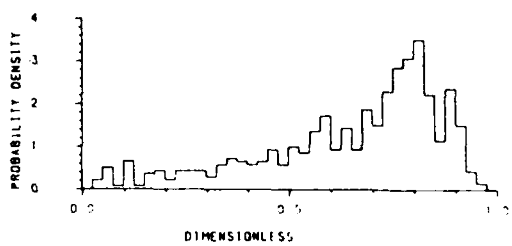


STATISTICAL PARAMETERS

NUMBER OF SAMPLE VALUES	557
ARITHMETIC MEAN VALUE, M1	7.890-01
STANDARD DEVIATION, S	1.608-01
COEFFICIENT OF VARIATION, M1/S	4.907+00
COEFFICIENT OF SKEWNESS, K3/S**3	-1.942+00
COEFFICIENT OF EXCESS, K4/S**4	2.004+00
SECOND CENTRAL MOMENT, C2	2.585-02
THIRD CENTRAL MOMENT, C3	-6.488-03
FOURTH CENTRAL MOMENT, C4	3.398-03
FOURTH CUMULANT, K4=(C4-3*C2**2)	1.393-03
SECOND MOMENT ABOUT ZERO, M2	6.484-01
THIRD MOMENT ABOUT ZERO, M3	5.459-01
FOURTH MOMENT ABOUT ZERO, M4	4.673-01
MINIMUM VALUE	1.969-01
MAXIMUM VALUE	9.754-01

SPECTRAL WIDTH SQUARED, EPSILON**2

BALLAST CONDITIONS



STATISTICAL PARAMETERS

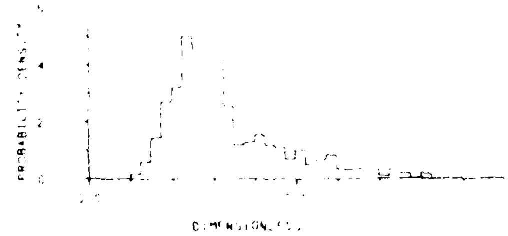
NUMBER OF SAMPLE VALUES	557
ARITHMETIC MEAN VALUE, M1	6.484-01
STANDARD DEVIATION, S	2.167-01
COEFFICIENT OF VARIATION, M1/S	2.992+00
COEFFICIENT OF SKEWNESS, K3/S**3	-9.858-01
COEFFICIENT OF EXCESS, K4/S**4	1.987-01
SECOND CENTRAL MOMENT, C2	4.697-02
THIRD CENTRAL MOMENT, C3	-1.003-02
FOURTH CENTRAL MOMENT, C4	7.056-03
FOURTH CUMULANT, K4=(C4-3*C2**2)	4.384-04
SECOND MOMENT ABOUT ZERO, M2	4.673-01
THIRD MOMENT ABOUT ZERO, M3	3.539-01
FOURTH MOMENT ABOUT ZERO, M4	2.761-01
MINIMUM VALUE	3.877-02
MAXIMUM VALUE	9.514-01

PERIOD RATIO, ALPHA-TP/TZ
FULL LOAD CONDITIONS

STATISTICAL PARAMETERS

NUMBER OF SAMPLE VALUES	473
ARITHMETIC MEAN VALUE, M1	5.618-01
STANDARD DEVIATION, S	1.115-01
COEFFICIENT OF VARIATION, M1/S	5.040+00
COEFFICIENT OF SKEWNESS, K3/S**3	7.790-01
COEFFICIENT OF EXCESS, K4/S**4	1.701-01
SECOND CENTRAL MOMENT, C2	1.243-02
THIRD CENTRAL MOMENT, C3	1.079-03
FOURTH CENTRAL MOMENT, C4	4.897-04
FOURTH CUMULANT, K4=(C4-3*C2**2)	2.628-05
SECOND MOMENT ABOUT ZERO, M2	3.281-01
THIRD MOMENT ABOUT ZERO, M3	1.993-01
FOURTH MOMENT ABOUT ZERO, M4	1.260-01
MINIMUM VALUE	3.266-01
MAXIMUM VALUE	9.014-01

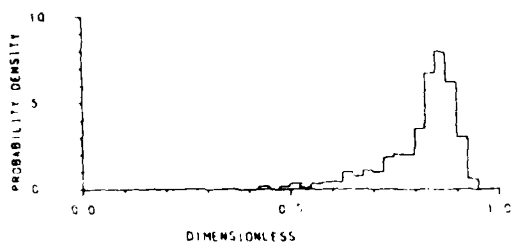
PERIOD RATIO SQUARED, ALPHA**2
FULL LOAD CONDITIONS



STATISTICAL PARAMETERS

NUMBER OF SAMPLE VALUES	473
ARITHMETIC MEAN VALUE, M1	3.281-01
STANDARD DEVIATION, S	1.358-01
COEFFICIENT OF VARIATION, M1/S	2.416+00
COEFFICIENT OF SKEWNESS, K3/S**3	1.206+00
COEFFICIENT OF EXCESS, K4/S**4	1.082+00
SECOND CENTRAL MOMENT, C2	1.845-02
THIRD CENTRAL MOMENT, C3	3.019-03
FOURTH CENTRAL MOMENT, C4	1.387-03
FOURTH CUMULANT, K4=(C4-3*C2**2)	3.677-04
SECOND MOMENT ABOUT ZERO, M2	1.260-01
THIRD MOMENT ABOUT ZERO, M3	5.648-02
FOURTH MOMENT ABOUT ZERO, M4	2.876-02
MINIMUM VALUE	1.067-01
MAXIMUM VALUE	8.125-01

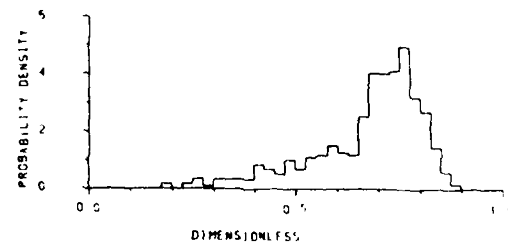
SPECTRAL WIDTH, EPSILON-SQRT(1-(TP/TZ)**2)
FULL LOAD CONDITIONS



STATISTICAL PARAMETERS

NUMBER OF SAMPLE VALUES	473
ARITHMETIC MEAN VALUE, M1	8.147-01
STANDARD DEVIATION, S	9.080-02
COEFFICIENT OF VARIATION, M1/S	8.972+00
COEFFICIENT OF SKEWNESS, K3/S**3	-1.547+00
COEFFICIENT OF EXCESS, K4/S**4	2.371+00
SECOND CENTRAL MOMENT, C2	8.245-03
THIRD CENTRAL MOMENT, C3	-1.159-03
FOURTH CENTRAL MOMENT, C4	3.652-04
FOURTH CUMULANT, K4=(C4-3*C2**2)	1.612-04
SECOND MOMENT ABOUT ZERO, M2	6.719-01
THIRD MOMENT ABOUT ZERO, M3	5.597-01
FOURTH MOMENT ABOUT ZERO, M4	4.699-01
MINIMUM VALUE	4.339-01
MAXIMUM VALUE	9.452-01

SPECTRAL WIDTH SQUARED, EPSILON**2
FULL LOAD CONDITIONS

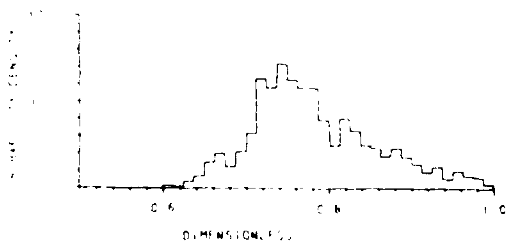


STATISTICAL PARAMETERS

NUMBER OF SAMPLE VALUES	473
ARITHMETIC MEAN VALUE, M1	6.719-01
STANDARD DEVIATION, S	1.350-01
COEFFICIENT OF VARIATION, M1/S	4.949+00
COEFFICIENT OF SKEWNESS, K3/S**3	-1.206+00
COEFFICIENT OF EXCESS, K4/S**4	1.884+00
SECOND CENTRAL MOMENT, C2	1.843-02
THIRD CENTRAL MOMENT, C3	-3.014-05
FOURTH CENTRAL MOMENT, C4	1.303-03
FOURTH CUMULANT, K4=(C4-3*C2**2)	3.682-04
SECOND MOMENT ABOUT ZERO, M2	4.699-01
THIRD MOMENT ABOUT ZERO, M3	3.379-01
FOURTH MOMENT ABOUT ZERO, M4	2.470-01
MINIMUM VALUE	1.875-01
MAXIMUM VALUE	8.934-01

FRACTION OF POSITIVE MAXIMA, A

ALL LOADING CONDITIONS

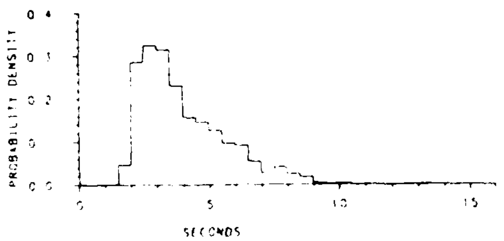


STATISTICAL PARAMETERS

NUMBER OF SAMPLE VALUES	1030
ARITHMETIC MEAN VALUE, M1	7.821-01
STANDARD DEVIATION, S	7.497-02
COEFFICIENT OF VARIATION, M1/S	1.043+01
COEFFICIENT OF SKEWNESS, K3/S**3	5.723-01
COEFFICIENT OF EXCESS, K4/S**4	-1.206-02
SECOND CENTRAL MOMENT, C2	5.620-03
THIRD CENTRAL MOMENT, C3	2.411-04
FOURTH CENTRAL MOMENT, C4	9.438-05
FOURTH CUMULANT, K4=(C4-3*C2**2)	-3.808-07
SECOND MOMENT ABOUT ZERO, M2	6.173-01
THIRD MOMENT ABOUT ZERO, M3	4.918-01
FOURTH MOMENT ABOUT ZERO, M4	3.956-01
MINIMUM VALUE	6.103-01
MAXIMUM VALUE	9.902-01

PERIOD OF POSITIVE MAXIMA, TP+ =TP/A

ALL LOADING CONDITIONS

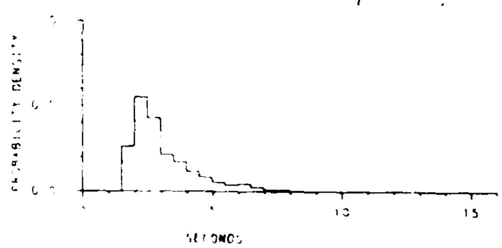


STATISTICAL PARAMETERS

NUMBER OF SAMPLE VALUES	1027
ARITHMETIC MEAN VALUE, M1	4.003+00
STANDARD DEVIATION, S	1.637+00
COEFFICIENT OF VARIATION, M1/S	2.443+00
COEFFICIENT OF SKEWNESS, K3/S**3	1.115+00
COEFFICIENT OF EXCESS, K4/S**4	1.195+00
SECOND CENTRAL MOMENT, C2	2.681+00
THIRD CENTRAL MOMENT, C3	4.894+00
FOURTH CENTRAL MOMENT, C4	2.979+01
FOURTH CUMULANT, K4=(C4-3*C2**2)	8.229+00
SECOND MOMENT ABOUT ZERO, M2	1.869+01
THIRD MOMENT ABOUT ZERO, M3	1.013+02
FOURTH MOMENT ABOUT ZERO, M4	6.203+02
MINIMUM VALUE	1.829+00
MAXIMUM VALUE	1.300+01

AV.PEAK PERIOD, TP

ALL LOADING CONDITIONS

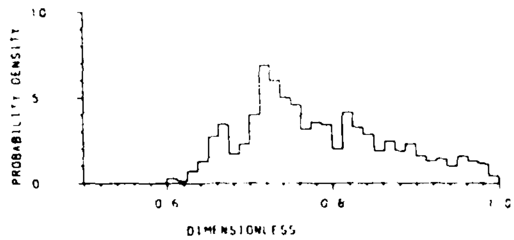


STATISTICAL PARAMETERS

NUMBER OF SAMPLE VALUES	1030
ARITHMETIC MEAN VALUE, M1	3.095+00
STANDARD DEVIATION, S	1.256+00
COEFFICIENT OF VARIATION, M1/S	2.465+00
COEFFICIENT OF SKEWNESS, K3/S**3	1.796+00
COEFFICIENT OF EXCESS, K4/S**4	4.689+00
SECOND CENTRAL MOMENT, C2	1.577+00
THIRD CENTRAL MOMENT, C3	3.556+00
FOURTH CENTRAL MOMENT, C4	1.912+01
FOURTH CUMULANT, K4=(C4-3*C2**2)	1.166+01
SECOND MOMENT ABOUT ZERO, M2	1.116+01
THIRD MOMENT ABOUT ZERO, M3	4.782+01
FOURTH MOMENT ABOUT ZERO, M4	2.452+02
MINIMUM VALUE	1.763+00
MAXIMUM VALUE	1.229+01

FRACTION OF POSITIVE MAXIMA A

BALLAST CONDITIONS

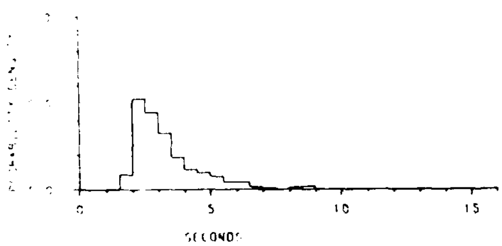


STATISTICAL PARAMETERS

NUMBER OF SAMPLE VALUES	557
ARITHMETIC MEAN VALUE, M1	7.831-01
STANDARD DEVIATION, S	8.809-02
COEFFICIENT OF VARIATION, M1/S	8.890+00
COEFFICIENT OF SKEWNESS, K3/S**3	4.689-01
COEFFICIENT OF EXCESS, K4/S**4	-5.487-01
SECOND CENTRAL MOMENT, C2	7.759-03
THIRD CENTRAL MOMENT, C3	3.205-04
FOURTH CENTRAL MOMENT, C4	1.476-04
FOURTH CUMULANT, K4=(C4-3*C2**2)	-3.303-05
SECOND MOMENT ABOUT ZERO, M2	6.210-01
THIRD MOMENT ABOUT ZERO, M3	4.989-01
FOURTH MOMENT ABOUT ZERO, M4	4.050-01
MINIMUM VALUE	6.103-01
MAXIMUM VALUE	9.902-01

PERIOD OF POSITIVE MAXIMA, TP+ =TP/A

BALLAST CONDITIONS

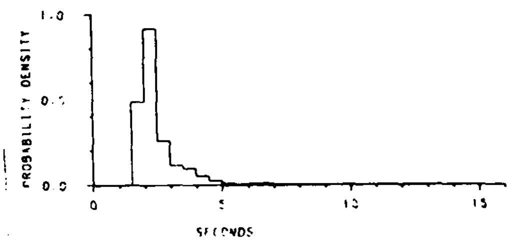


STATISTICAL PARAMETER

NUMBER OF SAMPLE VALUES	554
ARITHMETIC MEAN VALUE, M1	2.304+00
STANDARD DEVIATION, S	1.332+00
COEFFICIENT OF VARIATION, M1/S	2.480+00
COEFFICIENT OF SKEWNESS, K3/S**3	2.129+00
COEFFICIENT OF EXCESS, K4/S**4	7.212+00
SECOND CENTRAL MOMENT, C2	1.774+00
THIRD CENTRAL MOMENT, C3	5.031+00
FOURTH CENTRAL MOMENT, C4	3.215+01
FOURTH CUMULANT, K4=(C4-3*C2**2)	2.271+01
SECOND MOMENT ABOUT ZERO, M2	1.269+01
THIRD MOMENT ABOUT ZERO, M3	5.462+01
FOURTH MOMENT ABOUT ZERO, M4	3.331+02
MINIMUM VALUE	1.829+00
MAXIMUM VALUE	1.300+01

AV.PEAK PERIOD, TP

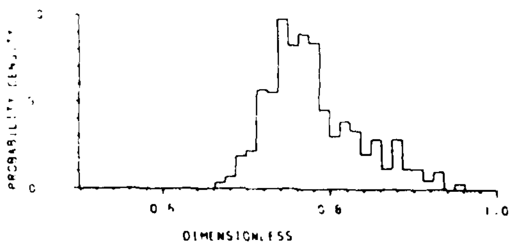
BALLAST CONDITIONS



STATISTICAL PARAMETERS

NUMBER OF SAMPLE VALUES	551
ARITHMETIC MEAN VALUE, M1	2.496+00
STANDARD DEVIATION, S	8.044-01
COEFFICIENT OF VARIATION, M1/S	3.102+00
COEFFICIENT OF SKEWNESS, K3/S**3	2.524+00
COEFFICIENT OF EXCESS, K4/S**4	8.020+00
SECOND CENTRAL MOMENT, C2	6.471-01
THIRD CENTRAL MOMENT, C3	1.313+00
FOURTH CENTRAL MOMENT, C4	4.614+00
FOURTH CUMULANT, K4=(C4-3*C2**2)	3.358+00
SECOND MOMENT ABOUT ZERO, M2	6.874+00
THIRD MOMENT ABOUT ZERO, M3	2.163+01
FOURTH MOMENT ABOUT ZERO, M4	8.052+01
MINIMUM VALUE	1.763+00
MAXIMUM VALUE	7.234+00

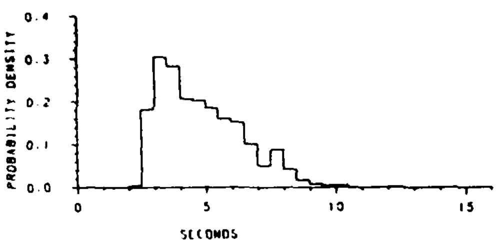
FRACTION OF POSITIVE MAXIMA, A
FULL LOAD CONDITIONS



STATISTICAL PARAMETERS

NUMBER OF SAMPLE VALUES	473
ARITHMETIC MEAN VALUE, M1	7.689-01
STANDARD DEVIATION, S	5.575-02
COEFFICIENT OF VARIATION, M1/S	1.401+01
COEFFICIENT OF SKEWNESS, K3/S**3	7.734-01
COEFFICIENT OF EXCESS, K4/S**4	2.379-01
SECOND CENTRAL MOMENT, C2	3.108-03
THIRD CENTRAL MOMENT, C3	1.341-04
FOURTH CENTRAL MOMENT, C4	3.120-05
FOURTH CUMULANT, K4=(C4-3*C2**2)	2.298-06
SECOND MOMENT ABOUT ZERO, M2	6.129-01
THIRD MOMENT ABOUT ZERO, M3	4.836-01
FOURTH MOMENT ABOUT ZERO, M4	[illegible]
MINIMUM VALUE	6.633-01
MAXIMUM VALUE	[illegible]

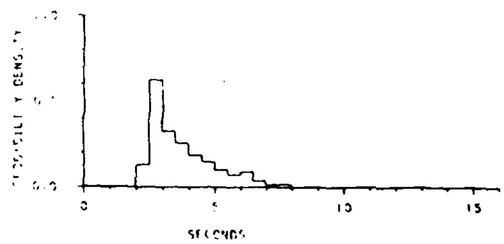
PERIOD OF POSITIVE MAXIMA, TP+ =TP/A
FULL LOAD CONDITIONS



STATISTICAL PARAMETERS

NUMBER OF SAMPLE VALUES	473
ARITHMETIC MEAN VALUE, M1	4.816+00
STANDARD DEVIATION, S	1.549+00
COEFFICIENT OF VARIATION, M1/S	3.038+00
COEFFICIENT OF SKEWNESS, K3/S**3	1.052-01
COEFFICIENT OF EXCESS, K4/S**4	-2.823-01
SECOND CENTRAL MOMENT, C2	2.511+00
THIRD CENTRAL MOMENT, C3	2.807+00
FOURTH CENTRAL MOMENT, C4	1.767+01
FOURTH CUMULANT, K4=(C4-3*C2**2)	-1.279+00
SECOND MOMENT ABOUT ZERO, M2	2.570+01
THIRD MOMENT ABOUT ZERO, M3	1.507+02
FOURTH MOMENT ABOUT ZERO, M4	9.577+02
MINIMUM VALUE	2.475+00
MAXIMUM VALUE	1.412+01

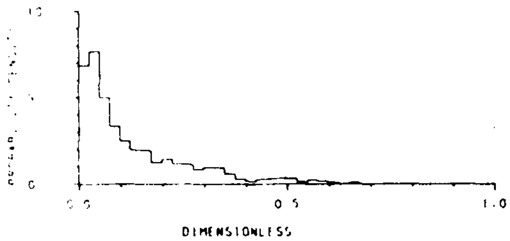
AV PEAK PERIOD, TP
FULL LOAD CONDITIONS



STATISTICAL PARAMETERS

NUMBER OF SAMPLE VALUES	473
ARITHMETIC MEAN VALUE, M1	3.731+00
STANDARD DEVIATION, S	1.216+00
COEFFICIENT OF VARIATION, M1/S	3.069+00
COEFFICIENT OF SKEWNESS, K3/S**3	1.131+00
COEFFICIENT OF EXCESS, K4/S**4	6.189-01
SECOND CENTRAL MOMENT, C2	1.478+00
THIRD CENTRAL MOMENT, C3	2.033+00
FOURTH CENTRAL MOMENT, C4	7.902+00
FOURTH CUMULANT, K4=(C4-3*C2**2)	1.308+00
SECOND MOMENT ABOUT ZERO, M2	[illegible]
THIRD MOMENT ABOUT ZERO, M3	[illegible]
FOURTH MOMENT ABOUT ZERO, M4	[illegible]
MINIMUM VALUE	[illegible]
MAXIMUM VALUE	[illegible]

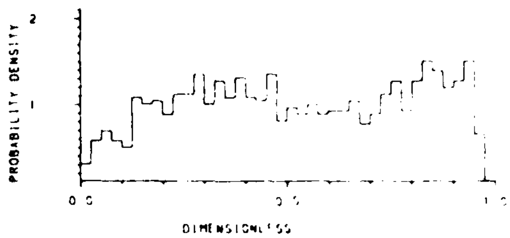
PEAK PERIOD DIVISION RATIO. (T -TS)/(TB-TS)
ALL LOADING CONDITIONS



STATISTICAL PARAMETERS

NUMBER OF SAMPLE VALUES	1030
ARITHMETIC MEAN VALUE, M1	1.301-01
STANDARD DEVIATION, S	1.314-01
COEFFICIENT OF VARIATION, M1/S	9.903-01
COEFFICIENT OF SKEWNESS, K3/S**3	1.558+00
COEFFICIENT OF EXCESS, K4/S**4	2.160+00
SECOND CENTRAL MOMENT, C2	1.726-02
THIRD CENTRAL MOMENT, C3	3.532-03
FOURTH CENTRAL MOMENT, C4	1.537-03
FOURTH CUMULANT, K4=(C4-3*C2**2)	6.432-04
SECOND MOMENT ABOUT ZERO, M2	3.416-02
THIRD MOMENT ABOUT ZERO, M3	1.245-02
FOURTH MOMENT ABOUT ZERO, M4	5.397-03
MINIMUM VALUE	[illegible]
MAXIMUM VALUE	[illegible]

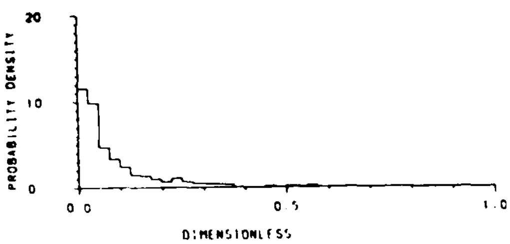
ZERO PERIOD DIVISION RATIO. (TZ-TS)/(TB-TS)
ALL LOADING CONDITIONS



STATISTICAL PARAMETERS

NUMBER OF SAMPLE VALUES	1030
ARITHMETIC MEAN VALUE, M1	5.245-01
STANDARD DEVIATION, S	2.712-01
COEFFICIENT OF VARIATION, M1/S	1.934+00
COEFFICIENT OF SKEWNESS, K3/S**3	-4.327-02
COEFFICIENT OF EXCESS, K4/S**4	-1.212+00
SECOND CENTRAL MOMENT, C2	7.356-02
THIRD CENTRAL MOMENT, C3	-8.633-04
FOURTH CENTRAL MOMENT, C4	9.674-03
FOURTH CUMULANT, K4=(C4-3*C2**2)	-6.559-03
SECOND MOMENT ABOUT ZERO, M2	3.486-01
THIRD MOMENT ABOUT ZERO, M3	2.590-01
FOURTH MOMENT ABOUT ZERO, M4	2.048-01
MINIMUM VALUE	1.244-02
MAXIMUM VALUE	9.985-01

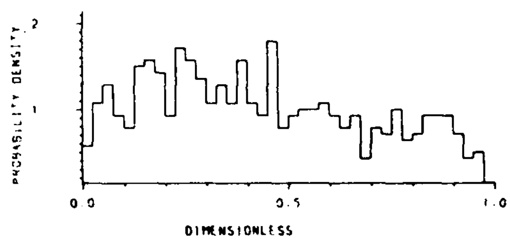
PEAK PERIOD DIVISION RATIO. (T -TS)/(TB-TS)
BALLAST CONDITIONS



STATISTICAL PARAMETERS

NUMBER OF SAMPLE VALUES	557
ARITHMETIC MEAN VALUE, M1	8.340-02
STANDARD DEVIATION, S	5.606-02
COEFFICIENT OF VARIATION, M1/S	6.686-01
COEFFICIENT OF SKEWNESS, K3/S**3	2.252+00
COEFFICIENT OF EXCESS, K4/S**4	5.934+00
SECOND CENTRAL MOMENT, C2	5.227-03
THIRD CENTRAL MOMENT, C3	1.996-03
FOURTH CENTRAL MOMENT, C4	7.606-04
FOURTH CUMULANT, K4=(C4-3*C2**2)	5.052-04
SECOND MOMENT ABOUT ZERO, M2	1.617-02
THIRD MOMENT ABOUT ZERO, M3	4.869-03
FOURTH MOMENT ABOUT ZERO, M4	1.947-03
MINIMUM VALUE	1.090-03
MAXIMUM VALUE	[illegible]

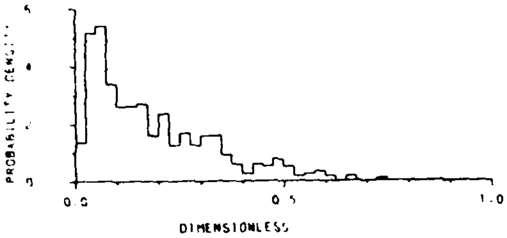
ZERO PERIOD DIVISION RATIO. (TZ-TS)/(TB-TS)
BALLAST CONDITIONS



STATISTICAL PARAMETERS

NUMBER OF SAMPLE VALUES	557
ARITHMETIC MEAN VALUE, M1	4.380-01
STANDARD DEVIATION, S	2.643-01
COEFFICIENT OF VARIATION, M1/S	1.661+00
COEFFICIENT OF SKEWNESS, K3/S**3	3.825-01
COEFFICIENT OF EXCESS, K4/S**4	-9.949-01
SECOND CENTRAL MOMENT, C2	6.983-02
THIRD CENTRAL MOMENT, C3	5.570-03
FOURTH CENTRAL MOMENT, C4	9.777-03
FOURTH CUMULANT, K4=(C4-3*C2**2)	-4.851-03
SECOND MOMENT ABOUT ZERO, M2	[illegible]
THIRD MOMENT ABOUT ZERO, M3	[illegible]
FOURTH MOMENT ABOUT ZERO, M4	1.370-01
MINIMUM VALUE	[illegible]
MAXIMUM VALUE	[illegible]

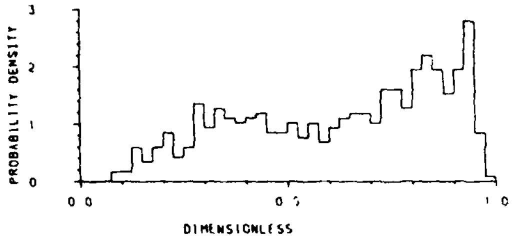
PEAK PERIOD DIVISION RATIO. (T -TS)/(TB-TS)
FULL LOAD CONDITIONS



STATISTICAL PARAMETERS

NUMBER OF SAMPLE VALUES	473
ARITHMETIC MEAN VALUE, M1	1.850-01
STANDARD DEVIATION, S	1.455-01
COEFFICIENT OF VARIATION, M1/S	1.272+00
COEFFICIENT OF SKEWNESS, K3/S**3	1.088+00
COEFFICIENT OF EXCESS, K4/S**4	6.359-01
SECOND CENTRAL MOMENT, C2	2.116-02
THIRD CENTRAL MOMENT, C3	3.348-03
FOURTH CENTRAL MOMENT, C4	1.628-03
FOURTH CUMULANT, K4=(C4-3*C2**2)	2.847-04
SECOND MOMENT ABOUT ZERO, M2	5.539-02
THIRD MOMENT ABOUT ZERO, M3	2.137-02
FOURTH MOMENT ABOUT ZERO, M4	9.575-03
MINIMUM VALUE	1.358-02
MAXIMUM VALUE	7.271-01

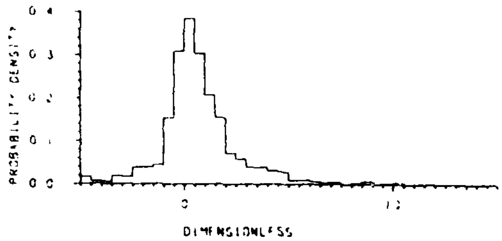
ZERO PERIOD DIVISION RATIO. (TZ-TS)/(TB-TS)
FULL LOAD CONDITIONS



STATISTICAL PARAMETERS

NUMBER OF SAMPLE VALUES	473
ARITHMETIC MEAN VALUE, M1	6.254-01
STANDARD DEVIATION, S	2.434-01
COEFFICIENT OF VARIATION, M1/S	2.567+00
COEFFICIENT OF SKEWNESS, K3/S**3	-3.831-01
COEFFICIENT OF EXCESS, K4/S**4	-1.091+00
SECOND CENTRAL MOMENT, C2	5.925-02
THIRD CENTRAL MOMENT, C3	-5.526-03
FOURTH CENTRAL MOMENT, C4	6.701-03
FOURTH CUMULANT, K4=(C4-3*C2**2)	-3.831-03
SECOND MOMENT ABOUT ZERO, M2	4.502-01
THIRD MOMENT ABOUT ZERO, M3	3.303-01
FOURTH MOMENT ABOUT ZERO, M4	2.645-01
[illegible] VALUE	8.660-02
[illegible] VALUE	5.779-01

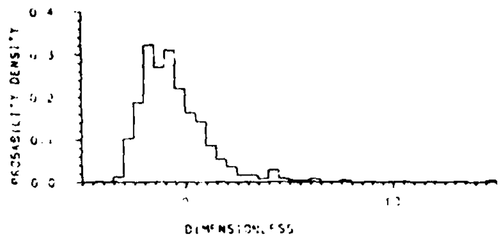
NORM.EXTREME OF TOTAL STRESS. HALF RANGE
ALL LOADING CONDITIONS



STATISTICAL PARAMETERS

NUMBER OF SAMPLE VALUES	725
ARITHMETIC MEAN VALUE, M1	6.646-01
STANDARD DEVIATION, S	1.783+00
COEFFICIENT OF VARIATION, M1/S	3.727-01
COEFFICIENT OF SKEWNESS, K3/S**3	6.967-01
COEFFICIENT OF EXCESS, K4/S**4	3.933+00
SECOND CENTRAL MOMENT, C2	3.177+00
THIRD CENTRAL MOMENT, C3	5.949+00
FOURTH CENTRAL MOMENT, C4	7.007+01
FOURTH CUMULANT, K4=(C4-3*C2**2)	3.975+01
SECOND MOMENT ABOUT ZERO, M2	3.616+00
THIRD MOMENT ABOUT ZERO, M3	1.055+01
FOURTH MOMENT ABOUT ZERO, M4	8.864+01
MINIMUM VALUE	-5.926+00
MAXIMUM VALUE	1.043+01

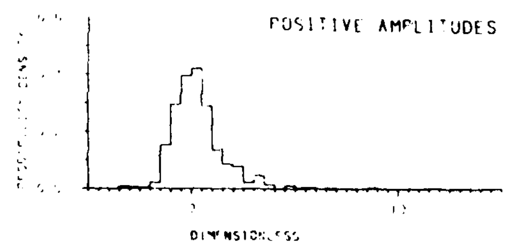
NORM.EXTREME OF SPRINGING STRESS. HALF RANGE
ALL LOADING CONDITIONS



STATISTICAL PARAMETERS

NUMBER OF SAMPLE VALUES	492
ARITHMETIC MEAN VALUE, M1	-4.953-01
STANDARD DEVIATION, S	2.008+00
COEFFICIENT OF VARIATION, M1/S	-2.467-01
COEFFICIENT OF SKEWNESS, K3/S**3	5.284+00
COEFFICIENT OF EXCESS, K4/S**4	5.926+01
SECOND CENTRAL MOMENT, C2	4.031+00
THIRD CENTRAL MOMENT, C3	4.277+01
FOURTH CENTRAL MOMENT, C4	1.012+03
FOURTH CUMULANT, K4=(C4-3*C2**2)	9.630+02
SECOND MOMENT ABOUT ZERO, M2	4.268+00
THIRD MOMENT ABOUT ZERO, M3	3.632+01
FOURTH MOMENT ABOUT ZERO, M4	9.253+02
MINIMUM VALUE	-3.253+00
MAXIMUM VALUE	2.573+01

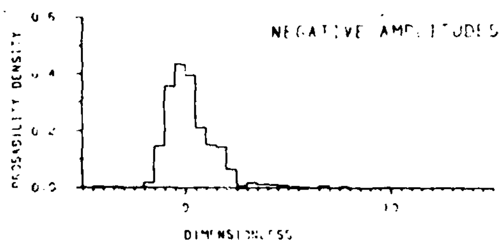
NORM.EXTREME OF BENDING STRESS.
ALL LOADING CONDITIONS



STATISTICAL PARAMETERS

NUMBER OF SAMPLE VALUES	560
ARITHMETIC MEAN VALUE, M1	3.208-01
STANDARD DEVIATION, S	1.263+00
COEFFICIENT OF VARIATION, M1/S	2.540-01
COEFFICIENT OF SKEWNESS, K3/S**3	1.509+00
COEFFICIENT OF EXCESS, K4/S**4	5.435+00
SECOND CENTRAL MOMENT, C2	1.596+00
THIRD CENTRAL MOMENT, C3	3.042+00
FOURTH CENTRAL MOMENT, C4	2.149+01
FOURTH CUMULANT, K4=(C4-3*C2**2)	1.384+01
SECOND MOMENT ABOUT ZERO, M2	1.698+00
THIRD MOMENT ABOUT ZERO, M3	4.587+00
FOURTH MOMENT ABOUT ZERO, M4	2.618+01
MINIMUM VALUE	-3.418+00
MAXIMUM VALUE	6.633+00

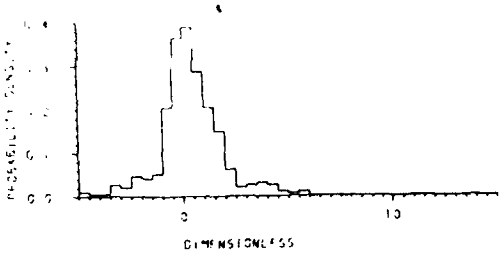
NORM.EXTREME OF BENDING STRESS.
ALL LOADING CONDITIONS



STATISTICAL PARAMETERS

NUMBER OF SAMPLE VALUES	560
ARITHMETIC MEAN VALUE, M1	2.670-01
STANDARD DEVIATION, S	1.247+00
COEFFICIENT OF VARIATION, M1/S	2.141-01
COEFFICIENT OF SKEWNESS, K3/S**3	1.699+00
COEFFICIENT OF EXCESS, K4/S**4	6.095+00
SECOND CENTRAL MOMENT, C2	1.555+00
THIRD CENTRAL MOMENT, C3	3.264+00
FOURTH CENTRAL MOMENT, C4	2.200+01
FOURTH CUMULANT, K4=(C4-3*C2**2)	1.475+01
SECOND MOMENT ABOUT ZERO, M2	1.624+00
THIRD MOMENT ABOUT ZERO, M3	4.505+00
FOURTH MOMENT ABOUT ZERO, M4	2.595+01
MINIMUM VALUE	-4.169+00
MAXIMUM VALUE	7.915+00

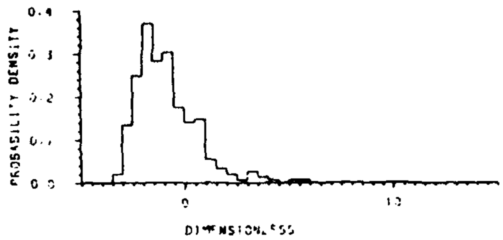
NORM.EXTREME OF TOTAL STRESS. HALF RANGE
BALLAST CONDITIONS



STATISTICAL PARAMETERS

NUMBER OF SAMPLE VALUES	389
ARITHMETIC MEAN VALUE, M1	3.749-01
STANDARD DEVIATION, S	1.496+00
COEFFICIENT OF VARIATION, M1/S	2.506-01
COEFFICIENT OF SKEWNESS, K3/S**3	1.878-01
COEFFICIENT OF EXCESS, K4/S**4	2.381+00
SECOND CENTRAL MOMENT, C2	2.239+00
THIRD CENTRAL MOMENT, C3	3.613-01
FOURTH CENTRAL MOMENT, C4	2.698+01
FOURTH CUMULANT, K4=(C4-3*C2**2)	1.194+01
SECOND MOMENT ABOUT ZERO, M2	2.374+00
THIRD MOMENT ABOUT ZERO, M3	2.922+00
FOURTH MOMENT ABOUT ZERO, M4	2.987+01
MINIMUM VALUE	-[illegible].471+00
MAXIMUM VALUE	[illegible]

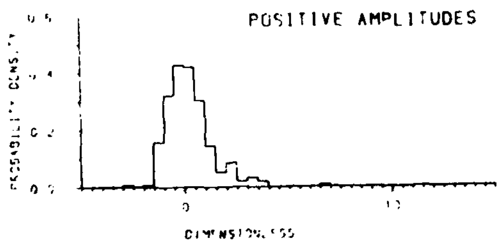
NORM.EXTREME OF SPRINGING STRESS. HALF RANGE
BALLAST CONDITIONS



STATISTICAL PARAMETERS

NUMBER OF SAMPLE VALUES	297
ARITHMETIC MEAN VALUE, M1	-8.458-01
STANDARD DEVIATION, S	1.447+00
COEFFICIENT OF VARIATION, M1/S	-5.844-01
COEFFICIENT OF SKEWNESS, K3/S**3	1.303+00
COEFFICIENT OF EXCESS, K4/S**4	2.410+00
SECOND CENTRAL MOMENT, C2	2.095+00
THIRD CENTRAL MOMENT, C3	3.950+00
FOURTH CENTRAL MOMENT, C4	2.374+01
FOURTH CUMULANT, K4=(C4-3*C2**2)	1.057+01
SECOND MOMENT ABOUT ZERO, M2	2.803+00
THIRD MOMENT ABOUT ZERO, M3	-2.005+00
FOURTH MOMENT ABOUT ZERO, M4	1.962+01
MINIMUM VALUE	-3.253+00
MAXIMUM VALUE	5.533+00

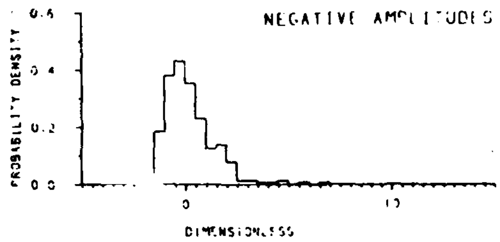
NORM.EXTREME OF BENDING STRESS.
BALLAST CONDITIONS



STATISTICAL PARAMETERS

NUMBER OF SAMPLE VALUES	305
ARITHMETIC MEAN VALUE, M1	2.459-01
STANDARD DEVIATION, S	1.118+00
COEFFICIENT OF VARIATION, M1/S	2.199-01
COEFFICIENT OF SKEWNESS, K3/S**3	1.305+00
COEFFICIENT OF EXCESS, K4/S**4	3.814+00
SECOND CENTRAL MOMENT, C2	1.250+00
THIRD CENTRAL MOMENT, C3	1.825+00
FOURTH CENTRAL MOMENT, C4	1.065+01
FOURTH CUMULANT, K4=(C4-3*C2**2)	5.962+00
SECOND MOMENT ABOUT ZERO, M2	1.307+00
THIRD MOMENT ABOUT ZERO, M3	2.735+00
FOURTH MOMENT ABOUT ZERO, M4	1.271+01
MINIMUM VALUE	-2.914+00
MAXIMUM VALUE	6.601+00

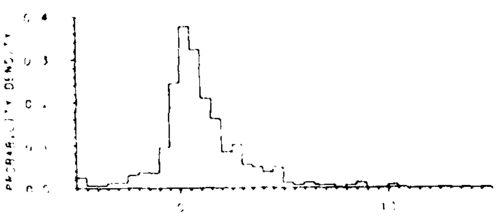
NORM.EXTREME OF BENDING STRESS.
BALLAST CONDITIONS



STATISTICAL PARAMETERS

NUMBER OF SAMPLE VALUES	305
ARITHMETIC MEAN VALUE, M1	[illegible].342-01
STANDARD DEVIATION, S	1.197+00
COEFFICIENT OF VARIATION, M1/S	1.956-01
COEFFICIENT OF SKEWNESS, K3/S**3	1.603+00
COEFFICIENT OF EXCESS, K4/S**4	4.309+00
SECOND CENTRAL MOMENT, C2	1.434+00
THIRD CENTRAL MOMENT, C3	2.751+00
FOURTH CENTRAL MOMENT, C4	1.502+01
FOURTH CUMULANT, K4=(C4-3*C2**2)	8.857+00
SECOND MOMENT ABOUT ZERO, M2	1.444+00
THIRD MOMENT ABOUT ZERO, M3	3.732+00
FOURTH MOMENT ABOUT ZERO, M4	1.768+01
MINIMUM VALUE	-1.757+00
MAXIMUM VALUE	6.871+00

NORM.EXTREME OF TOTAL STRESS. HALF RANGE
FULL LOAD CONDITIONS

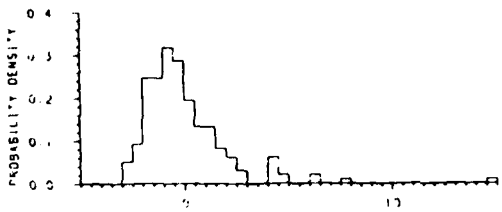


DIMENSIONLESS

STATISTICAL PARAMETERS

NUMBER OF SAMPLE VALUES	335
ARITHMETIC MEAN VALUE, M1	9.990-01
STANDARD DEVIATION, S	2.017+00
COEFFICIENT OF VARIATION, M1/S	4.953-01
COEFFICIENT OF SKEWNESS, K3/S**3	7.673-01
COEFFICIENT OF EXCESS, K4/S**4	3.591+00
SECOND CENTRAL MOMENT, C2	4.067+00
THIRD CENTRAL MOMENT, C3	6.293+00
FOURTH CENTRAL MOMENT, C4	1.093+02
FOURTH CUMULANT, K4=(C4-3*C2**2)	5.340+01
SECOND MOMENT ABOUT ZERO, M2	[illegible]
THIRD MOMENT ABOUT ZERO, M3	1.938+01
FOURTH MOMENT ABOUT ZERO, M4	1.576+02
MINIMUM VALUE	-5.815+00
MAXIMUM VALUE	1.043+01

NORM.EXTREME OF SPRINGING STRESS. HALF RANGE
FULL LOAD CONDITIONS

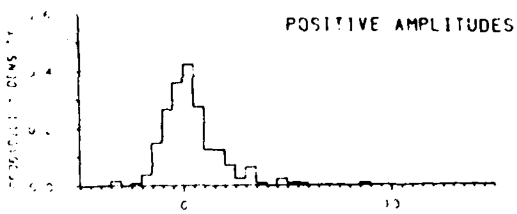


DIMENSIONLESS

STATISTICAL PARAMETERS

NUMBER OF SAMPLE VALUES	195
ARITHMETIC MEAN VALUE, M1	3.855-02
STANDARD DEVIATION, S	2.556+00
COEFFICIENT OF VARIATION, M1/S	1.508-02
COEFFICIENT OF SKEWNESS, K3/S**3	5.745+00
COEFFICIENT OF EXCESS, K4/S**4	5.183+01
SECOND CENTRAL MOMENT, C2	6.532+00
THIRD CENTRAL MOMENT, C3	9.590+01
FOURTH CENTRAL MOMENT, C4	2.339+03
FOURTH CUMULANT, K4=(C4-3*C2**2)	2.211+03
SECOND MOMENT ABOUT ZERO, M2	6.500+00
THIRD MOMENT ABOUT ZERO, M3	9.469+01
FOURTH MOMENT ABOUT ZERO, M4	2.305+03
MINIMUM VALUE	-2.993+00
MAXIMUM VALUE	2.573+01

NORM.EXTREME OF BENDING STRESS.
FULL LOAD CONDITIONS

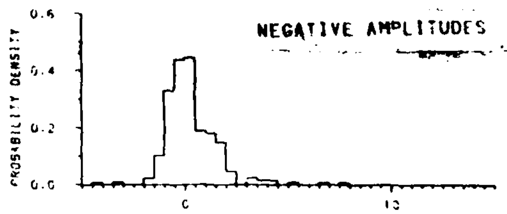


DIMENSIONLESS

STATISTICAL PARAMETERS

NUMBER OF SAMPLE VALUES	255
ARITHMETIC MEAN VALUE, M1	4.105-01
STANDARD DEVIATION, S	1.415+00
COEFFICIENT OF VARIATION, M1/S	2.901-01
COEFFICIENT OF SKEWNESS, K3/S**3	1.535+00
COEFFICIENT OF EXCESS, K4/S**4	5.450+00
SECOND CENTRAL MOMENT, C2	2.001+00
THIRD CENTRAL MOMENT, C3	4.346+00
FOURTH CENTRAL MOMENT, C4	3.384+01
FOURTH CUMULANT, K4=(C4-3*C2**2)	2.183+01
SECOND MOMENT ABOUT ZERO, M2	2.162+00
THIRD MOMENT ABOUT ZERO, M3	6.802+00
FOURTH MOMENT ABOUT ZERO, M4	4.228+01
MINIMUM VALUE	-3.418+00
MAXIMUM VALUE	8.633+00

NORM.EXTREME OF BENDING STRESS.
FULL LOAD CONDITIONS

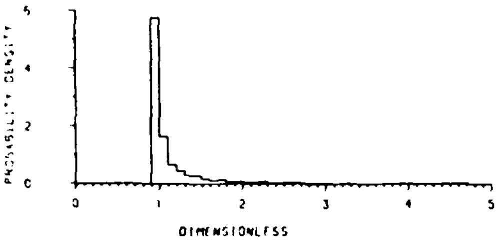


DIMENSIONLESS

STATISTICAL PARAMETERS

NUMBER OF SAMPLE VALUES	255
ARITHMETIC MEAN VALUE, M1	3.062-01
STANDARD DEVIATION, S	1.306+00
COEFFICIENT OF VARIATION, M1/S	2.346-01
COEFFICIENT OF SKEWNESS, K3/S**3	1.759+00
COEFFICIENT OF EXCESS, K4/S**4	7.526+00
SECOND CENTRAL MOMENT, C2	1.704+00
THIRD CENTRAL MOMENT, C3	3.782+00
FOURTH CENTRAL MOMENT, C4	3.850+01
FOURTH CUMULANT, K4=(C4-3*C2**2)	2.186+01
SECOND MOMENT ABOUT ZERO, M2	1.792+00
THIRD MOMENT ABOUT ZERO, M3	5.438+00
FOURTH MOMENT ABOUT ZERO, M4	3.570+01
MINIMUM VALUE	-4.167+00
MAXIMUM VALUE	7.915+00

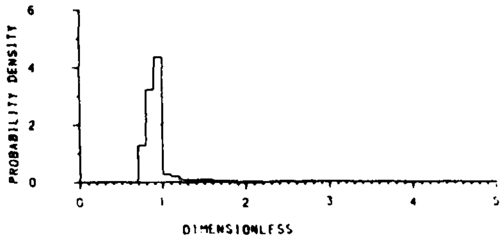
FATIGUE FACTOR, LAMBDA' M=3
ALL LOADING CONDITIONS



STATISTICAL PARAMETERS

NUMBER OF SAMPLE VALUES	1027
ARITHMETIC MEAN VALUE, M1	1.135+00
STANDARD DEVIATION, S	3.420-01
COEFFICIENT OF VARIATION, M1/S	3.319+00
COEFFICIENT OF SKEWNESS, K3/S**3	3.413+00
COEFFICIENT OF EXCESS, K4/S**4	1.361+01
SECOND CENTRAL MOMENT, C2	1.170-01
THIRD CENTRAL MOMENT, C3	1.363-01
FOURTH CENTRAL MOMENT, C4	2.273-01
FOURTH CUMULANT, K4=(C4-3*C2**2)	1.863-01
SECOND MOMENT ABOUT ZERO, M2	1.405+00
THIRD MOMENT ABOUT ZERO, M3	1.997+00
FOURTH MOMENT ABOUT ZERO, M4	3.408+00
MINIMUM VALUE	9.180-01
MAXIMUM VALUE	3.629+00

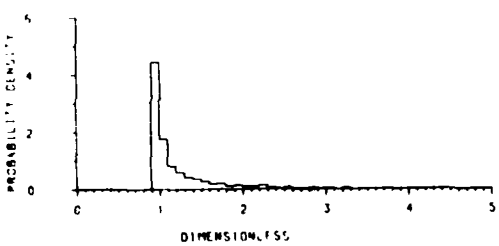
FATIGUE FACTOR, LAMBDA' M=4
ALL LOADING CONDITIONS



STATISTICAL PARAMETERS

NUMBER OF SAMPLE VALUES	1027
ARITHMETIC MEAN VALUE, M1	9.659-01
STANDARD DEVIATION, S	2.865-01
COEFFICIENT OF VARIATION, M1/S	3.371+00
COEFFICIENT OF SKEWNESS, K3/S**3	4.271+00
COEFFICIENT OF EXCESS, K4/S**4	2.159+01
SECOND CENTRAL MOMENT, C2	8.210-02
THIRD CENTRAL MOMENT, C3	1.005-01
FOURTH CENTRAL MOMENT, C4	1.657-01
FOURTH CUMULANT, K4=(C4-3*C2**2)	1.455-01
SECOND MOMENT ABOUT ZERO, M2	1.015+00
THIRD MOMENT ABOUT ZERO, M3	1.239+00
FOURTH MOMENT ABOUT ZERO, M4	1.841+00
MINIMUM VALUE	7.221-01
MAXIMUM VALUE	[illegible]

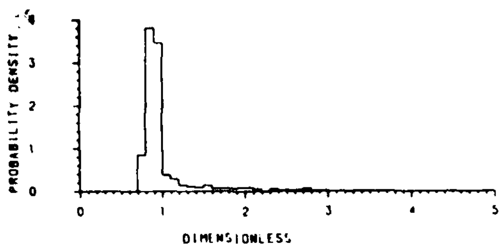
FATIGUE FACTOR, LAMBDA' M=3
BALLAST CONDITIONS



STATISTICAL PARAMETERS

NUMBER OF SAMPLE VALUES	554
ARITHMETIC MEAN VALUE, M1	1.227+00
STANDARD DEVIATION, S	4.313-01
COEFFICIENT OF VARIATION, M1/S	2.846+00
COEFFICIENT OF SKEWNESS, K3/S**3	2.513+00
COEFFICIENT OF EXCESS, K4/S**4	6.747+00
SECOND CENTRAL MOMENT, C2	1.860-01
THIRD CENTRAL MOMENT, C3	2.016-01
FOURTH CENTRAL MOMENT, C4	3.373-01
FOURTH CUMULANT, K4=(C4-3*C2**2)	2.330-01
SECOND MOMENT ABOUT ZERO, M2	1.692+00
THIRD MOMENT ABOUT ZERO, M3	2.733+00
FOURTH MOMENT ABOUT ZERO, M4	5.264+00
MINIMUM VALUE	9.180-01
MAXIMUM VALUE	3.629+00

FATIGUE FACTOR, LAMBDA' M=4
BALLAST CONDITIONS

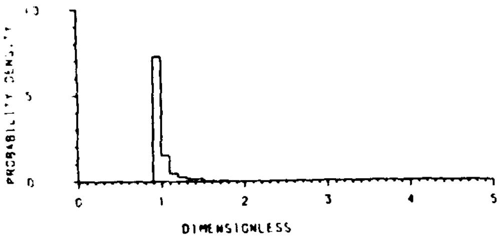


STATISTICAL PARAMETERS

NUMBER OF SAMPLE VALUES	554
ARITHMETIC MEAN VALUE, M1	1.018+00
STANDARD DEVIATION, S	3.721-01
COEFFICIENT OF VARIATION, M1/S	2.735+00
COEFFICIENT OF SKEWNESS, K3/S**3	3.225+00
COEFFICIENT OF EXCESS, K4/S**4	1.114+01
SECOND CENTRAL MOMENT, C2	1.384-01
THIRD CENTRAL MOMENT, C3	1.661-01
FOURTH CENTRAL MOMENT, C4	2.710-01
FOURTH CUMULANT, K4=(C4-3*C2**2)	2.135-01
SECOND MOMENT ABOUT ZERO, M2	1.179+00
THIRD MOMENT ABOUT ZERO, M3	1.641+00
FOURTH MOMENT ABOUT ZERO, M4	2.871+00
MINIMUM VALUE	7.309-01
MAXIMUM VALUE	[illegible]

FATIGUE FACTOR, LAMBDA' M=3

FULL LOAD CONDITIONS

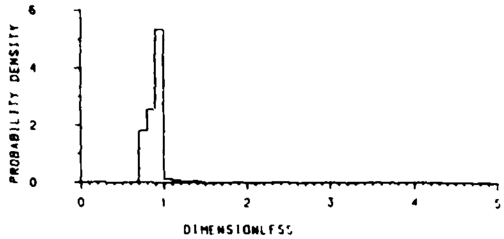


STATISTICAL PARAMETERS

NUMBER OF SAMPLE VALUES	473
ARITHMETIC MEAN VALUE, M1	1.027+00
STANDARD DEVIATION, S	1.210-01
COEFFICIENT OF VARIATION, M1/S	8.485+00
COEFFICIENT OF SKEWNESS, K3/S**3	3.560+00
COEFFICIENT OF EXCESS, K4/S**4	1.393+01
SECOND CENTRAL MOMENT, C2	1.465-02
THIRD CENTRAL MOMENT, C3	6.313-03
FOURTH CENTRAL MOMENT, C4	3.646-03
FOURTH CUMULANT, K4=(C4-3*C2**2)	3.002-03
SECOND MOMENT ABOUT ZERO, M2	1.069+00
THIRD MOMENT ABOUT ZERO, M3	1.135+00
FOURTH MOMENT ABOUT ZERO, M4	1.234+00
MINIMUM VALUE	9.273-01
MAXIMUM VALUE	1.754+00

FATIGUE FACTOR, LAMBDA' M=4

FULL LOAD CONDITIONS



STATISTICAL PARAMETERS

NUMBER OF SAMPLE VALUES	473
ARITHMETIC MEAN VALUE, M1	9.052-01
STANDARD DEVIATION, S	4.720-02
COEFFICIENT OF VARIATION, M1/S	5.313+00
COEFFICIENT OF SKEWNESS, K3/S**3	5.146-01
COEFFICIENT OF EXCESS, K4/S**4	4.480+00
SECOND CENTRAL MOMENT, C2	5.444-03
THIRD CENTRAL MOMENT, C3	8.399-04
FOURTH CENTRAL MOMENT, C4	6.676-04
FOURTH CUMULANT, K4=(C4-3*C2**2)	3.997-04
SECOND MOMENT ABOUT ZERO, M2	8.289-01
THIRD MOMENT ABOUT ZERO, M3	7.692-01
FOURTH MOMENT ABOUT ZERO, M4	7.214-01
MINIMUM VALUE	7.221-01
MAXIMUM VALUE	1.457+00

FILMED

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